

# TABLE OF CONTENTS

List of Figures.....	<i>xiii</i>
List of Tables.....	<i>xix</i>
Foreword.....	<i>xxiii</i>
Preface .....	<i>xxv</i>
Acknowledgments.....	<i>xxix</i>
<b>1</b>	<b>Introduction to Multi-Criteria Decision Making..... 1</b>
1.1	Multi-Criteria Decision Making: A General Overview .....
	1
1.2	Classification of MCDM Methods .....
	3
<b>2</b>	<b>Multi-Criteria Decision Making Methods ..... 5</b>
2.1	Background Information .....
	5
2.2	Description of Some MCDM Methods.....
	5
2.2.1	The WSM Method .....
	6
2.2.2	The WPM Method .....
	8
2.2.3	The AHP Method .....
	9
2.2.4	The Revised AHP Method.....
	11
2.2.5	The ELECTRE Method.....
	13
2.2.6	The TOPSIS Method.....
	18
<b>3</b>	<b>Quantification of Qualitative Data for MCDM Problems ..... 23</b>
3.1	Background Information .....
	23
3.2	Scales for Quantifying Pairwise Comparisons.....
	25
3.2.1	Scales Defined on the Interval $[9, 1/9]$ .....
	26
3.2.2	Exponential Scales.....
	28
3.2.3	Some Examples of the Use of Exponential Scales.....
	29
3.3	Evaluating Different Scales.....
	32
3.3.1	The Concepts of the RCP and CDP Matrices .....
	32
3.3.2	On The Consistency of CDP Matrices.....
	35
3.3.3	Two Evaluative Criteria .....
	43
3.4	A Simulation Evaluation of Different Scales .....
	44
3.5	Analysis of the Computational Results .....
	50

3.6	Conclusions.....	53
<b>4</b>	<b>Deriving Relative Weights from Ratio Comparisons .....</b>	<b>57</b>
4.1	Background Information .....	57
4.2	The Eigenvalue Approach.....	58
4.3	Some Optimization Approaches.....	60
4.4	Considering The Human Rationality Factor .....	61
4.5	First Extensive Numerical Example .....	65
4.6	Second Extensive Numerical Example .....	66
4.7	Average Error per Comparison for Sets of Different Size.....	67
4.8	Conclusions.....	72
<b>5</b>	<b>Deriving Relative Weights from Difference Comparisons .....</b>	<b>73</b>
5.1	Background Information .....	73
5.2	Pairwise Comparisons of Relative Similarity.....	76
5.2.1	Quantifying Pairwise Comparisons of Relative Similarity.....	76
5.2.2	Processing Pairwise Comparisons of Relative Similarity.....	77
5.2.3	An Extensive Numerical Example .....	79
5.3	Conclusions.....	85
<b>6</b>	<b>A Decomposition Approach for Evaluating Relative Weights Derived from Comparisons .....</b>	<b>87</b>
6.1	Background Information .....	87
6.2	Problem Description .....	88
6.3	Two Solution Approaches .....	91
6.3.1	A Simple Approach.....	91
6.3.2	A Linear Programming Approach.....	92
6.4	An Extensive Numerical Example .....	95
6.5	Some Computational Experiments.....	97
6.6	Analysis of the Computational Results .....	100
6.7	Conclusions.....	112
<b>7</b>	<b>Reduction of Pairwise Comparisons Via a Duality Approach.....</b>	<b>115</b>
7.1	Background Information .....	115
7.2	A Duality Approach for Eliciting Comparisons.....	116
7.3	An Extensive Numerical Example .....	120

7.3.1	Applying the Primal Approach.....	121
7.3.2	Applying the Dual Approach.....	122
7.4	Some Numerical Results for Problems of Different Sizes .....	124
7.5	Conclusions.....	128
<b>8</b>	<b>A Sensitivity Analysis Approach for MCDM Methods .....</b>	<b>131</b>
8.1	Background Information .....	131
8.2	Description of the Two Major Sensitivity Analysis Problems .....	133
8.3.	<b>Problem 1: Determining the Most Critical Criterion .....</b>	<b>135</b>
8.3.1	Definitions and Terminology.....	135
8.3.2	Some Theoretical Results in Determining the Most Critical Criterion.....	137
8.3.2.1	Case (i): Using the WSM or the AHP Method .....	137
8.3.2.2	An Extensive Numerical Example for the WSM Case .....	138
8.3.2.3	Case (ii): Using the WPM Method .....	142
8.3.2.4	An Extensive Numerical Example for the WPM Case.....	143
8.3.3	Some Computational Experiments.....	145
8.4	<b>Problem 2: Determining the Most Critical <math>a_{ij}</math> Measure of Performance.....</b>	<b>155</b>
8.4.1	Definitions and Terminology.....	155
8.4.2	Determining the Threshold Values $\hat{\delta}'_{i,j,k}$ .....	157
8.4.2.1	Case (i): When Using the WSM or the AHP Method.....	157
8.4.2.2	An Extensive Numerical Example When the WSM or the AHP Method is Used.....	158
8.4.2.3	Case (ii): When Using the WPM Method.....	161
8.4.2.4	An Extensive Numerical Example When the WPM Method is Used.....	161
8.5	Conclusions.....	165

	<b>Appendix to Chapter 8.....</b>	<b>167</b>
8.6	Calculation of the $\ddot{a}_{1,1,2}$ Quantity When the AHP or the WSM Method is Used .....	167
8.7	Calculation of the $\ddot{a}_{1,1,2}$ Quantity When the WPM Method is Used .....	169
8.8	Calculation of the $\hat{\delta}_{3,4,5}$ Quantity When the WSM Method is Used.....	170
8.9	Calculation of the $\hat{\delta}_{3,4,5}$ Quantity When the AHP Method is Used.....	171
8.10	Calculation of the $\hat{\delta}_{3,4,5}$ Quantity When the WPM Method is Used .....	174
<b>9</b>	<b>Evaluation of Methods for Processing a Decision Matrix and Some Cases of Ranking Abnormalities .....</b>	<b>177</b>
9.1	Background Information .....	177
9.2	Two Evaluative Criteria .....	177
9.3	Testing the Methods by Using the First Evaluative Criterion .....	179
9.4	Testing the Methods by Using the Second Evaluative Criterion .....	186
9.5	Analysis of the Computational Results .....	192
9.6	Evaluating the TOPSIS Method.....	194
9.7	Conclusions.....	197
<b>10</b>	<b>A Computational Evaluation of the Original and the Revised AHP.....</b>	<b>201</b>
10.1	Background Information .....	201
10.2	An Extensive Numerical Example .....	202
10.3	Some Computational Experiments.....	206
10.4	Conclusions.....	212
<b>11</b>	<b>More Cases of Ranking Abnormalities When Some MCDM Methods Are Used.....</b>	<b>213</b>
11.1	Background Information .....	213
11.2	Ranking Irregularities When Alternatives Are Compared Two at a Time .....	215
11.3	Ranking Irregularities When Alternatives Are Compared Two at a Time and Also as a Group .....	220
11.4	Some Computational Results .....	223

**Table of Contents**

xi

11.5	A Multiplicative Version of the AHP .....	228
11.6	Results from Two Real Life Case Studies.....	230
11.6.1	Comparative Ranking Analysis of the " <i>Bridge Evaluation</i> " Problem .....	230
11.6.2	Comparative Ranking Analysis of the " <i>Site Selection</i> " Problem.....	232
11.7	Conclusions.....	233
<b>12</b>	<b>Fuzzy Sets and Their Operations .....</b>	<b>235</b>
12.1	Background Information .....	235
12.2	Fuzzy Operations.....	236
12.3	Ranking of Fuzzy Numbers .....	238
<b>13</b>	<b>Fuzzy Multi-Criteria Decision Making .....</b>	<b>241</b>
13.1	Background Information .....	241
13.2	The Fuzzy WSM Method.....	242
13.3	The Fuzzy WPM Method.....	244
13.4	The Fuzzy AHP Method.....	245
13.5	The Fuzzy Revised AHP Method .....	247
13.6	The Fuzzy TOPSIS Method .....	248
13.7	Two Fuzzy Evaluative Criteria for Fuzzy MCDM Methods .....	250
13.7.1	Testing the Methods by Using the First Fuzzy Evaluative Criterion.....	251
13.7.2	Testing the Methods by Using the Second Fuzzy Evaluative Criterion.....	255
13.8	Computational Experiments.....	258
13.8.1	Description of the Computational Results .....	258
13.8.2	Analysis of the Computational Results .....	261
13.9	Conclusions.....	269
<b>14</b>	<b>Conclusions and Discussion for Future Research.....</b>	<b>263</b>
14.1	The Study of MCDM Methods: Future Trends .....	263
14.2	Lessons Learned.....	263
	<b>References .....</b>	<b>267</b>

**Subject Index..... 275**

**Author Index..... 283**

**About the Author ..... 289**