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Theory and Methodology

Evaluation of rankings with regard to the possible number of agreements and conflicts

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Abstract

An interesting problem in group decision analysis is how many different agreements can occur, or conversely disagreements may exist, between two or more different rankings of a set of alternatives. In this paper it is assumed that a reference ranking has been established for the set of alternatives. This reference ranking may represent the ranking of a high authority decision maker or be just a virtual ranking to be used in determining the discrepancy between pairs of rankings. Then, the problem examined here is to evaluate the number of possible rankings when the ranking method is the number of agreements with some reference ranking. The analysis presented here illustrates that this problem is not trivial and moreover, its simple context conceals complexity in its depth. The purpose of this paper is to provide an evaluation of the number of possible agreements in rankings given to a set of concepts, alternatives or ideas, by two or more decision makers. The number of possible agreements takes on the values $0, 1, 2, \ldots, n-2$, or n when n concepts are compared. This paper develops a recursive closed form formula for calculating the frequencies for the various numbers of agreements.

Keywords: Decision analysis; Ranking of alternatives; Multi-criteria decision making; Group decision making; Combinatorics; Conflict resolution

1. Introduction

There are many methods for determining the ranking of a set of alternatives in terms of a set of decision criteria. An overview of some widely used multi-criteria decision making (MCDM) methods is given by Chen and Hwang [1]. These methods include the analytic hierarchy process (AHP) [2], the weighted sum model (WSM) [3], the weighted product model (WPM) [4] and Miller and Starr [5], and TOPSIS [1] among others. A comparative examination of some of these MCDM methods was done by Triantaphyllou and Mann [6] and Lootsma [7].

There are many compelling reasons why one may be interested in evaluating the conflicts in different rankings. By better understanding and quantifying these conflicts, it is easier for multi-

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ple decision makers to reach an agreement or consensus by minimizing the required compromise. Further, some methods exist for the comparison of alternative rankings made on the same set of choices by different decision makers [8]. This paper is directed at the expansion of one of the methods discussed by Ray and Triantaphyllou [8].

In this paper it is assumed that there are a number of decision makers and they are interested in ranking n alternatives or concepts. Since it is very likely (if not certain) that they will have disagreements among themselves, a natural issue that is raised at this point is how to better understand the magnitude of their disagreements. This understanding, in turn, will better facilitate the results of the decision making process.

The importance of group decision making (GDM) becomes evident by the realization that in most cases decisions are made by groups as opposed by a single decision maker (see for instance, [9,10]). The need to improve group decision making has long been recognized by many researchers (e.g., [11,12]). For some more recent views in these issues, the interested reader may want to consult with the survey by Faure et al. [13]. One may argue that the applicability of the results of this paper in a single decision maker setting environment is limited because the number of alternatives is usually small (as Miller [14] stated humans can at most deal with approximately seven concepts at a given time). Thus, the main domain of applicability is GDM as opposed to a single decision maker environment.

This paper demonstrates that two rankings may have a number of disagreements which can occur according to certain frequencies, that is, some numbers of disagreements may be rarer than others. By understanding this issue in depth, the decision makers can have the opportunity to better comprehend the magnitude of their disagreements (or, equivalently, their agreements). Furthermore, it is assumed that the disagreements are all of equal importance. For instance, disagreeing on the top two concepts, has the same significance as disagreeing on the last two bottom concepts. This simplification can be dealt with in future research.

The problem examined in this paper is formally described as follows: Suppose that a given set of n alternatives, denoted as $A_1, A_2, A_3, \ldots, A_n$, are ranked differently by some decision makers. In this paper the terms alternatives, concepts, items or entities will be used interchangeably to denote the same concept. Moreover, some reference ranking is assumed. This reference ranking might be a target ranking, the ranking of the decision maker with the highest authority, or just a hypothetical ranking. Then, the central question examined here is: *How many possible rankings exist and of these possibilities how many deliver the same number of agreements with respect to some reference ranking*? The following definition will be used extensively in this paper.

Definition 1. A ranking is defined as an ordering on the set of alternatives.

For simplicity of the presentation style, we will use only the indexes to denote a specific ranking. For instance, if we deal with the four alternatives (or concepts) A_1, A_2, A_3, A_4, then one possible ranking is: $(2, 3, 4, 1)$, that is, alternative A_2 is the most preferred alternative, alternative A_3 is the next most preferred one, and so on and so forth.

Suppose that $(i_1, i_2, i_3, \ldots, i_n)$ and $(j_1, j_2, j_3, \ldots, j_n)$ are two possible rankings of n alternatives or concepts. That is, a ranking is an *n-tuple* of integers between 1 and n (with no two elements being identical). Section 2 introduces the terminology used in this paper. After the pertinent terminology is introduced, the general problem is discussed in Section 3. Section 4 discusses the main conclusions and findings of this paper.

2. Terminology

The following definitions were first introduced by Ray and Triantaphyllou [8] and are provided here for the purpose of clarification of any possible misunderstanding.

Definition 2. Rank: Is the position given to a specific alternative or concept within a given ranking.

Definition 3. Agreement: This state is a harmony of opinion that occurs when two different decision makers conclude that a given concept is of a specified rank.

If there are n concepts to be ranked, then the rank is an integer number between 1 and n. Rank of 1 is assumed to be the highest, while rank of n is considered to be the lowest. Moreover, it is assumed that no two concepts can have the same rank.

Definition 4 Conflict: The situation existing when decision makers fail to agree upon two or more concepts.

Generally, we are considering rankings of a set of ideas or concepts in some order, normally from the most important to the least important. For instance, for a 5-tuple of concepts designated by A_1, A_2, A_3, A_4 and A_5, one possible ranking would be (note that for simplicity we use only the indexes): $(5, 1, 4, 2, 3)$. This ranking really has no significance singly other than to say that it is the way a specific individual decision maker ranked the importance of the set in that particular manner. When the same set is ranked by a second individual, the ranking might be: $(3, 4, 1, 5, 2)$.

Next, consideration must be given to the fact that the two people have different opinions with regard to the importance of the concepts contained within the set $\{A_1, A_2, A_3, A_4, A_5\}$. These differences of opinions are termed as *conflicts* (Definition 4).

The main concept of the undertaken analysis is best illustrated via some simple cases. Namely, when the number of entities is equal to 1, 2, 3, and 4. First consideration is for the trite case of a 1-tuple. In this case there can be no disagreement. Obviously, there is only one possibility. This is summarized as follows:

Number of agreements	Frequency
Analysis for a 1-tuple (i.e., 1 ranking)	
1	1
0	0
Total	1

For the 2-tuple case only two positions are involved and there is either total agreement or total disagreement. The 2 (= 2!) possible rankings of the 2-tuple are: (1,2) and (2,1). Obviously, regardless of which one is the reference ranking, there are either two agreements or none (i.e., a single agreement cannot occur as it will be explained later). This is summarized as follows:

Number of agreements	Frequency
Analysis for a 2-tuple (i.e., 2 rankings)	
2	1
1	0
0	1
Total	2

Without loss of generality it can be assumed that the *correct*, or reference, ranking is: (1,2,3). All possible rankings of a 3-tuple are: (1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2) and (3,2,1). Thus, the numbers of agreements with the reference ranking are $3, 1, 1, 0, 0, 1$, respectively, for the six previous rankings. The total frequency of occurrences for the possible values of agreements are then 1 occurrence for 3 agreements, 0 for 2 agreements, 2 for 1 agreement and 3 for 1 agreement. The total number of outcomes sums to 6 (= 3!) which is the number of all possible rankings as summarized below.

Number of agreements	Frequency
Analysis for a 3-tuple (i.e., 6 rankings)	
3	1
2	0
1	3
0	2
Total	6

Similarly as above, for the 4-tuple there are 24 (= 4!) possible rankings. The following is an exhaustive enumeration of those possibilities:

$(1, 2, 3, 4)$ $(1, 2, 4, 3)$ $(1, 3, 2, 4)$ $(1, 3, 4, 2)$
$(2, 1, 3, 4)$ $(2, 1, 4, 3)$ $(2, 3, 1, 4)$ $(2, 3, 4, 1)$
$(3, 1, 2, 4)$ $(3, 1, 4, 2)$ $(3, 2, 1, 4)$ $(3, 2, 4, 1)$
$(4, 1, 2, 3)$ $(4, 1, 3, 2)$ $(4, 2, 1, 3)$ $(4, 2, 3, 1)$

$(1, 4, 2, 3)$ $(1, 4, 3, 2)$
$(2, 4, 1, 3)$ $(2, 4, 3, 1)$
$(3, 4, 1, 2)$ $(3, 4, 2, 1)$
$(4, 3, 1, 2)$ $(4, 3, 2, 1)$

The minimum number of agreements with a reference ranking (say, ranking (1,2,3,4)) is zero and the maximum number is 4 as summarized below.

Number of agreements	Frequency
For a 4-tuple (i.e., 24 rankings)	
4	1
3	0
2	6
1	8
0	9
Total	24

The above illustrative examples provide the motivation for generalizing when the number of entities to be compared is n, where n is some positive integer number. This is done in Section 3.

3. Generalization for any number of concepts n

The following corollary states a simple, but nevertheless fundamental, property on the number of agreements when n concepts are compared.

Corollary 1. *The minimum number of disagreements between any two rankings can never be equal to 1.*

Proof. To see why this is true consider two rankings, say $R_1 = (i_1, i_2, i_3, \ldots, i_n)$ and $R_2 = (j_1, j_2, j_3, \ldots, j_n)$, where i_y and j_y (for $y = 1, 2, 3, \ldots, n$) are numbers between 1 and n. Then, if the two rankings disagree in the Kth position ($n \geqslant K \geqslant 1$), the implication is that $i_K \neq j_K$. Therefore, they will *also* have to disagree in the Lth position where the Lth element in ranking R_1 is equal to j_K (i.e., $i_L = j_K$). Thus, there can never be just a single disagreement alone. For instance, if the two rankings are defined as: $R_1 = (1,2,3,4,5,6)$ and $R_2 = (1,2,5,6,3,4)$, then $K = 3$, $i_K = 3$, $j_K = 5$, $L = 5$, $i_L = 5$, and $j_L = 3$. □

Definitions 5 and 6 play a fundamental role in this generalization and are stated below.

Definition 5. Let $f_i^{(n)}$ (for $i = 0, 1, 2, \ldots, n$) be the number of occurrences, that is the frequency, of having i agreements with the reference ranking when n entities are ranked.

Definition 6. Let $\Phi_i^{(n)}$ (for $i = 0, 1, 2, \ldots, n$) be the adjusted frequency of having i agreements with the reference ranking when n entities are ranked and any one of them is replaced by an item which is not present in the reference ranking.

As an illustration of the frequency and adjusted frequency concepts consider the case of ranking three entities. Then as it was illustrated in Section 2, the following relations are true: $f_0^{(3)} = 2$, $f_1^{(3)} = 3$, $f_2^{(3)} = 0$, and $f_3^{(3)} = 1$. Moreover, the following relations are true for the case of the corresponding adjusted frequencies: $\Phi_0^{(3)} = 3$, $\Phi_1^{(3)} = 2$, $\Phi_2^{(3)} = 1$, and $\Phi_3^{(3)} = 0$. The latter numerical results can be easily confirmed with a simple exhaustive enumeration similar to the case of the $f_i^{(3)}$ frequencies. This new type of adjusted frequencies $\Phi_i^{(n)}$ is critical (as it will be shown next) in recursively determining the values of the original frequencies $f_i^{(n)}$. This is best summarized in Lemma 1 as follows.

Lemma 1. *The frequency $f_i^{(n)}$ of having i agreements with the reference ranking when n items are compared can be expressed as a function of the frequency $f_i^{(n-1)}$ and the adjusted frequency $\Phi_i^{(n-1)}$ as follows:*

$$f_i^{(n)} = \begin{cases} 1 & \text{if } i = n, \\ 0 & \text{if } i = n-1, \\ f_{i-1}^{(n)} + (n-1) \times \Phi_i^{(n-1)} & \text{for } i = n-2, \\ & n-3, \ldots, 2, 1, \\ (n-1) \times \Phi_0^{(n-1)} & \text{if } i = 0. \end{cases} \tag{1}$$

Proof. The relation $f_n^{(n)} = 1$ is true because there is only one ranking (i.e., the one identical to the reference ranking) which has n agreements with the reference ranking. The second relation (i.e., $f_{n-1}^{(n)} = 0$) is true because, according to Corollary 1, it is impossible to have only a single disagreement (or equivalently, $n - 1$ agreements).

To help validate the third relation of (1) consider a simple illustrative example, say when $n = 4$. All possible rankings for $n = 4$ are given earlier in Section 2. Out of these 24 (= 4!) rankings, six of them can be viewed as derived from the 6 (= 3!) rankings when three concepts are considered with the last rank being always equal to 4. The six previous rankings have frequencies $f_{i-1}^{(3)}$ for $i = 0, 1, 2, 3$. Therefore, in the general case, these rankings account for the $f_{i-1}^{(n-1)}$ term in the third relation of (1).

Next, it can be observed that the remaining 18 ($= 4! - 3! = 24 - 6$) rankings can be viewed as derived from three groups of rankings comprised of six rankings each. Each one of these rankings can be viewed as derived from the rankings when four concepts are ranked of which always one of them is equal to 4 and the rank of the last concept is equal to either 1, 2, or 3. Thus, each such group of rankings corresponds to the $\Phi_i^{(3)}$ adjusted frequencies for $i = 1, 2, 3$. At this point, for this particular illustrative example, the number of the groups in which the last rank is equal to either 1, 2, or 4, is equal to 3 ($= 4 - 1$). Therefore, in the general case when n concepts are compared, the rest of the $n! - n$ rankings can be viewed as derived from the rankings when $n - 1$ concepts are considered, of which any one of them is equal to n, augmented by the last rank which is equal to $1, 2, 3, \ldots, n - 1$, that is, the second term in the right-hand side of the third relation of (1) must be equal to $(n - 1) \times \Phi_i^{(n-1)}$. The above two terms are valid for $i = 1, 2, 3, \ldots, n - 2$.

Finally, the fourth (i.e., the last) case of relations (1) is true because when $i = 0$, then only the second term of the right-hand side of the third case remains valid. Thus, the truth of all cases in relations (1) is proved. □

Lemma 2 provides the recursive relations for calculating the $\Phi_i^{(n)}$ values.

Lemma 2. *The adjusted frequency $\Phi_i^{(n)}$ of having i agreements with the reference ranking when n items are considered, can be determined recursively as follows:*

$$\Phi_i^{(n)} = \begin{cases} 0 & \text{if } i = n, \\ 1 & \text{if } i = n - 1, \\ f_i^{(n)} + (f_{i+1}^{(n)} \times (i + 1) & \\ \quad -f_i^{(n)} \times i)/n & \text{for } i = n - 1, \\ & n - 2, \ldots, 2, 1, 0, \\ f_0^{(n)} + (f_1^{(n)}/n) & \text{if } i = 0. \end{cases} \tag{2}$$

Proof. It can be easily observed that when a single item (or entity) from the list of n items is discarded, then the number of rankings with i agreements with the reference ranking will change as a result of two factors. First, some of the rankings which before had $i + 1$ agreements, will now have i agreements (i.e., they will lose one agreement as a result of discarding the single item). Let N_1 denote the number of the new rankings with i agreements. Then, it can be easily verified that this number is given by the following relation:

$$N_1 = (f_{i+1}^{(n)} \times (i + 1))/n. \tag{3a}$$

The above relation (3a) follows easily from the observation that since the original number of rankings with i agreements is equal to $f_i^{(n)}$, then the total number of such agreements is equal to the product $f_{i+1}^{(n)} \times (i + 1)$. Therefore, since one item of n is dropped, the number of new rankings with i agreements will *increase* by the one-nth (i.e., $1/n$) of the previous product. Thus, relation (3a) is derived.

In an analogous manner it can be proved that the number of existing rankings with i agreements will be *reduced* by the amount N_2, given by relation (3b).

$$N_2 = (f_i^{(n)} \times i)/n. \tag{3b}$$

Therefore, from relations (3a) and (3b), relation (4) is derived.

$$\Phi_i^{(n)} = f_i^{(n)} + N_1 - N_2$$

or

$$\Phi_i^{(n)} = f_i^{(n)} + (f_{i+1}^{(n)} \times (i + 1) - f_i^{(n)} \times i)/n \quad \text{for } i = n - 1, n - 2, \ldots, 2, 1, 0. \tag{4}$$

From the above definition of the $\Phi_i^{(n)}$ values it follows that relation (5) is always true for any value of n.

$$\Phi_n^{(n)} = 0. \tag{5}$$

Moreover, from relation (5) the following two special case relations (6) and (7) follow:

$$\Phi_{n-1}^{(n)} = (f_n^{(n)} \times n - f_{n-1}^{(n)} \times (n - 1))/n$$

or

$$\Phi_{n-1}^{(n)} = 1. \tag{6}$$

$$\Phi_0^{(n)} = f_0^{(n)} + (f_1^{(n)} \times 1 - f_0^{(n)} \times 0)/n$$

or

$$\Phi_0^{(n)} = f_0^{(n)} + (f_1^{(n)} \times 1/n) = f_0^{(n)} + (f_1^{(n)}/n). \tag{7}$$

Relation (6) is always true because from Lemma 1 we have $f_{n-1}^{(n)} = 0$, and also $f_n^{(n)} = 1$. Relation (7) is obvious. Thus, the validity of Lemma 2 is proved. □

Lemma 1, in conjunction with Lemma 2, is next used to calculate the frequencies of having $0, 1, 2, \ldots, n$ agreements when n entities are compared (i.e., the $f_i^{(n)}$ values) in a recursive manner. This is best described in terms of Theorem 1.

Theorem 1. *The frequency $f_i^{(n)}$ of having i agreements with the reference ranking when n items are compared can be expressed recursively as follows:*

$$f_i^{(n)} = \begin{cases} 1 & \text{if } i = n, \\ 0 & \text{if } i = n-1, \\ f_{i-1}^{(n)} + (n-1) \times f_i^{(n-1)} + (i+1) \times f_{i+1}^{(n-1)} \\ \quad -i \times f_i^{(n-1)} & \text{for } i = n-2, n-3, \ldots, 2, 1, \\ f_1^{(n-1)} + (n-1) \times f_0^{(n-1)} & \text{if } i = 0. \end{cases} \tag{8}$$

Proof. The first relations of (8) are identical to the first two relations of (1) of Lemma 1. The third and fourth relations are easily derived when the $\Phi_i^{(n-1)}$ and $\Phi_0^{(n-1)}$ values are substituted with their equivalent expressions from Lemma 2. □

Next, Table 1 depicts the various $f_i^{(n)}$ values for different n values. Table 2 presents the previous results as percentages of the total number of possible rankings (which, of course, is equal to $n!$). A study of these two tables yields some interesting insights. One can notice the oscillation of the 0 and 1 agreement values as n increases. It is also noticeable that the percentages appear to approach a limit with increasing n.

It should be clarified here that in Table 2 the entries with the ε symbol indicate that this entry corresponds to a very small decimal value. These values can be determined as the ones in the rest of the entries. For instance, the entry (10,5) corresponds to the case in which there are $n = 10$ items to be compared and the number of agreements is equal to 6. For this case, the corresponding percentage is (see also Table 1): $\varepsilon = 5.21 \times 10^{-4}$ (= 1,890/10!).

Table 1
Number of agreements for various n values

Number of items n	Number of agreements										
	10	9	8	7	6	5	4	3	2	1	0
1										1	0
2									1	0	1
3								1	0	3	2
4							1	0	6	8	9
5						1	0	10	20	45	44
6					1	0	15	40	135	264	265
7				1	0	21	70	315	924	1 855	1 854
8			1	0	28	112	630	2 464	7 420	14 832	14 833
9		1	0	36	168	1 134	5 544	22 260	66 744	133 497	133 496
10	1	0	45	240	1 890	11 088	55 650	222 480	667 485	1 334 960	1 334 961

Table 2.
Number of agreements for various n values as percentages of all possibilities

Number of items n	Number of Agreements										
	10	9	8	7	6	5	4	3	2	1	0
1										1	0
2									0.5	0	0.5
3								0.1667	0	0.5	0.333333
4							0.0417	0	0.25	0.333333	0.375
5						0.0083	0	0.0833	0.1667	0.375	0.366667
6					ε	0	0.0208	0.0556	0.1875	0.366667	0.368056
7				ε	0	0.0042	0.0139	0.0625	0.1833	0.368056	0.367857
8			ε	0	ε	0.0028	0.0156	0.0611	0.1840	0.367857	0.367882
9		ε	0	ε	ε	0.0031	0.0153	0.0613	0.1839	0.367882	0.367879
10	ε	0	ε	ε	ε	0.0031	0.0153	0.0613	0.1839	0.367879	0.367879

4. Concluding remarks

When more than one decision maker is involved in ranking a set of n alternatives or concepts, it is unlikely, if not impossible, that all of them will agree in all rankings. Thus, an inherited issue is to enhance their understanding of the implications of their differences as much as possible. For instance, suppose that five alternatives or concepts (i.e., when $n = 5$) are considered and two decision makers have ranked them as $(2, 3, 1, 4, 5)$ and $(2, 5, 3, 4, 1)$, respectively. Therefore, the number of their agreements is equal to 2.

From Table 1 it can be seen that the total number of rankings which have two agreements with some reference ranking (in this case with the ranking of the first or the second decision maker) is equal to 20. Since the total number of all possible rankings is equal to 120 (i.e., 5!), this represents 16.67% of all possibilities (see also Table 2). If the number of agreements was equal to 3, then the previous two numbers would be 10 and 8.33%, respectively. It is also obvious that if the two decision makers try to compromise more, then the number of agreements may change to 5 (i.e., reach absolute agreement) and never to be equal to 4 (i.e., $n - 1$).

These numerical results are not obvious to the decision makers. However, they illuminate another dimension of the group decision making process. Thus, they can be used to better guide the various decision makers to better understand the size of their differences and the impact of increasing or reducing the number of their agreements.

The main contribution of this paper is best summarized in Theorem 1 and some numerical results are provided in Tables 1 and 2. Future research efforts in this exciting subject could focus on generalizing the concept of agreement when one wishes to somehow prioritize the importance of agreeing in the top alternatives vs. when agreeing on the bottom (and thus less critical) alternatives.

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