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AN EXAMPLE FOR DEALING WITH THE IMPRECISENESS OF FUTURE CASH FLOWS DURING THE SELECTION OF ECONOMIC ALTERNATIVES

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The correct estimation of cash flows is crucial during the selection of economic alternatives. Often, such estimations are imprecise due to the limited knowledge about future events that may affect the cash flows. These estimations are usually secured either by pure human intuition, statistical methodologies, or by combinations of these. The risk of an alternative can be measured easily because cash flow estimations are assumed to be normally distributed and thus they rely on the Central Limit Theorem. This paper presents an approach of how to deal with the imprecision of future cash flows that does not rely on the above theorem. We illustrate this approach by representing the cash flows as triangular fuzzy numbers. Although the example presented here considers that the cash flows have some symmetric imprecision or vagueness about a most promising value, a more realistic situation can be modeled by using asymmetric vagueness on the cash flows.

Significance: This paper illustrates an application to deal with the imprecision of estimated cash flows that does not rely on the Central Limit Theorem.

Keywords: Fuzzy decision-making, ranking of fuzzy numbers, internal rate of return, weighted method, Chang's method, Kaufmann and Gupta's method.

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1. INTRODUCTION

The accurate estimation of future cash flows for the selection of economic alternatives is a crucial task for decision-makers because an overestimation or underestimation may result in the selection of an unprofitable alternative (Fabrycky *et al.*, 1998). This estimation becomes crucial because it is often the result of estimating future income and cost data, such as future sales, selling prices, labor costs, interest rates, etc. Usually, cash flows are only informal estimations because the knowledge a decision-maker has about future events decreases as the time progresses in the planning horizon (Bussey and Echenbach, 1992). Moreover, they are only informal estimations because the relation of new economic alternatives with past experience is limited (Fabrycky *et al.*, 1998) and (Pike and Dobbin, 1986). Consequently, because cash estimations are hard to be accurately predicted, decision-makers often find it easier to express them by using linguistic expressions (Chui and Park, 1994). This paper presents an approach of how to deal with the estimation of future cash flows when they are expressed linguistically.

The estimation of cash flows is the result of qualified opinions about an economic outcome (Pike and Dobbind, 1986). Usually, these opinions are based on either pure human intuition, empirical evidence, the output of sophisticated mathematical models, or a combination of these. In practice, decision-makers measure their level of confidence on these estimations by assigning them a probabilistic value, and the "rule of thumb" they follow is: the higher this probabilistic value is, the better the estimation is. Traditionally, this level of confidence is the guide for selecting one of the following three decision-making methodologies to analyze the estimated data about the cash flows: decisions under certainty, decisions under risk, and decisions under uncertainty (Taha, 1997).

When decision-makers assign a complete level of confidence to the predicted cash flows, they have an absolute assurance that the projected estimations will coincide with the actual outcomes (Pike and Dobbind, 1986), (Bussey and Eschenbach, 1992), and (Taha, 1997). This is called decision-making under certainty because the outcome of an alternative can be confidently established. A bank's promise to pay a 4.5% APR on a 90-day CD is an example that illustrates a complete confidence on a future outcome. The performance measures frequently used for the selections of economic alternatives under certainty are the

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net present value (NPV), the internal rate of return (IRR), and the incremental internal rate of return (IIRR) (Chan, 1993), (Bussey and Echenbach, 1992), and (Fabrycky *et al.*, 1998).

Contrary, when the level of confidence on the estimated cash flows is partial, decision-makers recognize that the projected cash flows are estimated with an insufficient knowledge about some of the cash flows' elements (Pike and Dobbind, 1986). In this instance, decision-makers are said to make decisions under risk, and their main concern is to minimize the risk of making the wrong decision. Three methods are commonly used to address these types of decisions. In the first method, Fabrycky *et al.* (1998) indicate that the risk of an alternative is assessed by the magnitude of the cash flow deviation from the expected value. They suggest the utilization of the variance of the cash flows to measure this risk according to the following rule: The larger the variance, the larger the risk is. In the second method, Mao (1970) suggests that "although the deviations from the mean are favored by businessmen, it is only the 'downside' risk (the negative deviation) that is considered in the decision process." He suggested the semivariance (*SV*) to measure this downside risk as follows:

$$SV = \sum_{j=1}^{k} P_j (E_j \ \overline{E})^2. \qquad \ldots \qquad (1)$$

Where, P_j is the probability of occurrence of event j, the subscript j indicates all the values of E that are less than the expected value _, and k is the number of outcomes that are less than the expected value. In the third method, Levi and Sarnot (1982) suggest the utilization of the coefficient of variation (the standard deviation divided by the expected value) to compare the risk of alternatives when their outcomes are drastically different. The criteria for selecting the best alternative under risk are also based on the NPV, IRR, and IIRR criteria. However, this time the cash flows are expressed in terms of expected values (Chan, 1993), (Bussey and Echenbach, 1992), and (Fabrycky *et al.*, 1998).

The third type of decision making (i.e., under uncertainty) occurs when decision-makers consider either that they do not have meaningful knowledge even to assign a probability value to the estimated data (Pike and Dobbind, 1986) and (Taha, 1997), or that they are "unwilling to make those estimations because the event involves such things as natural disasters, a depression, or a bankruptcy" (Fabrycky *et al.*, 1998). In this case, an alternative A_k $(1 \le k \le t)$ is selected depending on which event e_j $(j = 1, 2, 3, ..., n)$ takes place in the future. For instance, if event e_3 occurs, then alternative A_1 may be more attractive than the remaining t - 1 alternatives. On the other hand, if event e_1 takes place, A_1 may be the least attractive alternative.

The interaction of alternatives and future events is called the "payoff matrix", which is used to summarize the possible future events over which decision-makers do not have control. The decision criteria often used for selecting alternatives from the payoff matrix are the Laplace Rule, Maximin Rule, Maximax Rule, Hurwicz Rule, and Minimax Regret Value (Holloway, 1979) and (Taha, 1997).

The purpose of this paper is to present the decision maker with an application approach of how to deal with the imprecision of cash flows, specially when they are vaguely defined. This application is based on the Fuzzy Set Theory introduced by (Zadeh, 1965). Zadeh indicates that "the natural language humans use to express their thoughts is often vaguely or imprecisely defined." The phrase "*a tall individual*" is an example of a concept that is imprecisely defined because depending on the standard used, an individual can be either tall or short. Traditionally, if the height 6' 4" is defined as the standard to determine the tallness of an individual, then all individuals under 6' 4" are said to be short. Otherwise, they are said to be tall. Under Zadeh's observations, however, an individual can be assigned a *degree of tallness* with respect to a preestablished standard. For example, if this standard is defined by the set of heights in the range 5' 6" and 6' 4", then a 5' 9" individual will be 0.91 (5' 9" ÷ 6' 4") tall with respect to the upper limit of this range. Similarly, a 6' 3" individual will be 0.98 (6' 3" ÷ 6' 4") tall. In the same vein, if the height of an individual is either 3' 4"or 7' 5", then he/she is said to be 0.0 tall because his/her height is off the range that defines tallness.

The theory presented in Dubois and Prade (1979,1983) provides the tools to represent linguistic statements such as "*a tall individual* " into mathematical operations. This theory has also been extended to deal with the vagueness of the future cash flows (see, for example, (Chui and Park, 1994) and (Buckley, 1987,1992)). To illustrate this concept, consider the statement "a future income of \$500,000±10%." This statement can have several interpretations. It can be interpreted as an income in the range [\$450,000 to \$550,000]. It can also represent a normally distributed income with mean μ = \$500,000 and σ = \$50,000. Alternatively, it can represent a Triangular Probability Distribution with typical value of \$500,000, and lower and upper limits of \$450,000 and \$550,000, respectively. Or, by using the operations in Dubois and Prade (1979,1983), such an expression can also be modeled as a triangular fuzzy number (TFN) (to be described later) with a most promising value m = \$500,000 and left and right spreads L = \$450,000 and R = \$550,000, respectively (Pohjola and Turunen, 1990). In the example presented in this paper, the estimated cash flows are modeled as TFN numbers.

The remaining of this paper is organized as follows. The next section introduces the concept of TFN numbers. The Third Section presents an overview of the internal rate of return, which is the decision criterion used in this paper for selecting economic alternatives. The Fourth Section describes four methods for ranking economic alternatives. The Fifth Section presents an example that illustrates the utilization of TFNs to model the cash flows of two mutually exclusive alternatives. Finally, this paper concludes with a summary section.

2. TRIANGULAR FUZZY NUMBERS

A triangular fuzzy number (TFN) M is a subset of real numbers whose membership function $\mu_M \in [0,1]$ is defined as follows (Dubois and Prade, 1979):

$$\mu_M(x) = \begin{cases} \frac{x-L}{m-L}, & for\ L \leq x \leq m, \\ \frac{x-R}{m-R}, & for\ m \leq x \leq R, \\ 0, & otherwise. \end{cases} \quad \cdots \quad (2)$$

Where m is the most promising value, and L and R are the left and right spreads (the smallest and largest values M can take). Often a TFN is expressed by its "LR-representation" (Dubois and Prade, 1979), which for TFN $M = (m, L, R)$. Figure 1 illustrates a TFN.

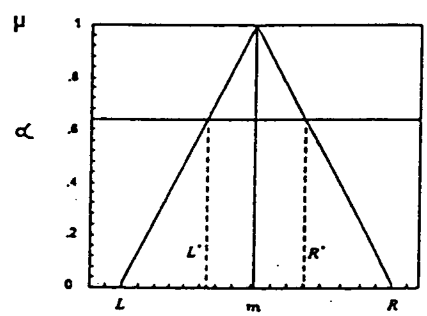

Figure 1. A Triangular Fuzzy Number (TFN).

The α symbol (also called α-*level*) is used to define different membership levels in the number (subset) M; it takes values from the range [0,1]. The α-*level* defines this membership by identifying the subset of values that M can only take. For instance, when α-*level* = 0, the membership in M is maximum because M can be any number in the range $L \leq M \leq R$. Contrary, when α-*level* = 1, the membership in M is unique because $M = m = L = R$. Notice that in this case, M is a deterministic number. Figure 1 illustrates the subset of values $L' \leq M \leq R'$ when α-*level* = 0.64.

In Dubois and Prade (1979), the usual arithmetic operations were extended to suit for TFNs. These operations are called extended operations. Equations (3) through Equation (5) illustrate the extended difference between the TFNs I and C.

$$F_m^{(\alpha=1)} = I_m^{(\alpha=1)} - C_m^{(\alpha=1)}, \quad \cdots \quad (3)$$

$$F_L^{(\alpha=0)} = I_L^{(\alpha=0)} - C_R^{(\alpha=0)}, \quad \cdots \quad (4)$$

$$F_R^{(\alpha=0)} = I_R^{(\alpha=0)} - C_L^{(\alpha=0)}. \quad \cdots \quad (5)$$

These values $F_m^{(\alpha=1)}$, $F_L^{(\alpha=0)}$, and $F_R^{(\alpha=0)}$ correspond to the *LR* representation of the TFN number *F*. That is, $F = (F_m, F_L, F_R)$. The superscript ($\alpha = x$) denotes the *α-level* at which the operation takes place. In Dubois and Prade (1979), the above three operations are conveniently represented as $F = I \_ C$, where the operator _ denotes the extended difference.

3. INTERNAL RATE OF RETURN

Under the traditional internal rate of return (IRR) criterion, the cash flows in period *t* (for $t = 0, \ldots, N$) are discounted at a rate i^* that makes the cash flows' net present value, $P(i^*)$, equal to zero. Equation 6 defines the net present value NPV of a series of future cash flows. Formally, the rate i^* must satisfy

$$P(i^*) = \sum_{t=0}^{N} (I_t C_t) \bullet (1+i^*)^t = 0. \quad \ldots \qquad (6)$$

In this equation, I_t and C_t are the incomes and costs (i.e., cash flows), respectively, in period *t*. For practical purposes of Eq. (6), the net cash flows are often sought to form the following pattern $I_{t^-} C_t < 0$, for $t = 0, \ldots, k$, and $I_{t^-} C_t \geq 0$, for $t = k+1, \ldots, N$ (Bussey and Eschenbach, 1992). Thus, when the cash flows form such a pattern, Eq. (6) has a unique solution for i^* in the range $-1 < i^* < \infty$, and for practical purposes in the range $0 < i^* < \infty$. A negative i^* would suggest the no recuperation of the original investment during the project's life (Chan, 1993).

It is important to notice that when the cash flows I_t and C_t (for $t = 1, \ldots, N$) are represented as TFNs, then the zero on the right-hand side of Eq. (6) can take two forms: It can be either a crisp zero (i.e., traditional zero), or it can be a TFN zero (which might be different from the traditional zero). In this paper, the case of crisp zero was selected because according to Pohjola and Turunen (1990), "it is the most familiar approach." By following this approach, the solution of Eq. (6) can also be characterized by a TFN of the internal rate of return i^*. Example 1 illustrates the solution of Eq. (6) when the cash flows are modeled as TFN numbers.

EXAMPLE 1. Compute the i^* value for the TFN cash flows given in Table 1. The second and third columns of this table represent the TFNs for the costs, *C*, and incomes, *I*, respectively. Columns fourth through sixth were obtained by applying Eqs. (3), (4), and (5), respectively. The last row shows the TFN i^*.

Table 1. Computation of i^* when Cost and Incomes are expressed as TFN Numbers.

t	*C*-	*I*-	$F_m^{(\alpha=1)}$	$F_L^{(\alpha=0)}$	$F_R^{(\alpha=0)}$
0	(7, 6, 8)	(0, 0, 0)	-7	-8	-6
1	(2, 2, 4)	(6, 5, 7)	4	1	5
2	(2, 2, 4)	(6, 5, 7)	4	1	5
		i(%)*	9.4‡	-58‡	42‡

‡ Obtained by solving Eq. (6).
-Data are illustrated in *LR*-representation.
The $F_x^{(\alpha=y)}$ values were obtained by solving Eqs. 3 through Eq. 5.

Thus, in this case $i^* = (9.4\%, -58\%, 42\%)$ indicates that the most likely IRR (i.e., when $\alpha = 1$) is 9.4%; whereas for the worst case scenario (i.e., when $\alpha = 0$), i^* may take any value from the range $-58\% \leq i^* \leq 42\%$.

4. RANKING OF ECONOMIC ALTERNATIVES

In the following subsections, two sets of criteria for the ranking of economic alternatives are presented. The traditional or crisp IRR criterion is presented first. Then, three criteria for ranking TFN numbers (i.e., i^* rates) are described.

4.1 The Crisp IRR Criterion

Under the crisp IRR criterion, alternatives are first ranked in descending order according to their i^* values. Then, the alternative with the largest i^* value is selected as the most economically attractive alternative (Bussey and Eschenbach, 1992) and (Chan, 1993).

4.2 The Fuzzy Criteria

Several methods have been proposed for ranking TFN numbers. For the case of cash flows, Chui and Park (1994) suggest the utilization of methods that are not tedious and require simple mathematical calculations. By following this suggestion, the three methods considered in this paper are the Weighted Method (Chui and Park, 1994), Chang's Method (Chang, 1981), and Kaufmann and Gupta's Method (Kaufmann and Gupta, 1988). Other ranking criteria and some fuzzy multi-criteria decision-making methods are found in (Dubois and Prade, 1983), (Chen, 1985), and (Triantaphyllou and Lin, 1996).

The Weighted Method. Under this method each alternative is first evaluated by using the following expression:

$$E_j = W_{1,j}\left(\frac{m_j + L_j + R_j}{3}\right) + W_{2,j}\, m_j, \quad for\ j = 1, 2, 3, \ldots \quad \ldots \quad (7)$$

Where E_j is the evaluation of the jth alternative, and $w_{1,j}$ and $w_{2,j}$ are two weights to denote the importance of the two terms on the right-hand side of the equation. According to Chui and Park (1994), "a common practice is to make $w_{1,j} = 1$ and $w_{2,j} = 0.3$ if the parameter m_j is important, otherwise $w_{1,j} = w_{2,j} = 0.1$." The ranking criterion for this method also consists of ordering in a descending manner the value E_j, (for $j = 1, 2, 3, \ldots$) of all the alternatives. The alternative with the largest E_j value is selected as the dominant (preferred) alternative.

Chang's Method. According to this method, the dominant alternative is the one with the largest mathematical expectation, E_j, which is defined by:

$$E_j = \frac{(R_j - L_j)(m_j + L_j + R_j)}{6}, \quad for\ j = 1, 2, 3, \ldots \quad \ldots \quad (8)$$

Kaufmann and Gupta's Method. This is a three-step ranking method. In this method, the dominance of each alternative is determined by the value of the following three priority indexes:

1. The ordinary number $N_j = \frac{(L_j + 2m_j + R_j)}{4}, \quad for\ j = 1, 2, 3, \ldots$
2. The most promising value m_j (for $j = 1, 2, 3, \ldots$) of each alternative.
3. The range $L_j - R_j$ (for $j = 1, 2, 3, \ldots$) of each alternative.

This dominance is determined as follows. First, the alternative with the largest ordinary number N_j is the dominant one (i.e., the preferred alternative so far). However, if some alternatives have the same largest ordinary number, then the dominant alternative is the one with the largest promising value m_j. Furthermore, if the largest ordinary number and the largest promising value are also the same for some alternatives, then the dominant alternative is decided by selecting the alternative with the larger range $L_j - R_j$. The following example illustrates the application of these three ranking methods.

EXAMPLE 2 (adapted from Triantaphyllou and Lin 1996). Let the outcomes of alternatives $\tilde{a}_1 = (0.4, 0.2, 0.6)$ and $\tilde{a}_2 = (0.7, 0.4, 0.9)$ be two TFNs. Table 2 shows the ranking of these alternatives by using the above fuzzy methods. The ranking of all three methods in the above table indicates that alternative $\tilde{a}_2$ is preferred to alternative $\tilde{a}_1$. It is interesting to notice that although these three methods ranked alternatives $\tilde{a}_1$ and $\tilde{a}_2$ consistently, Buckley's method, which was used in (Triantaphyllou and Lin, 1996), delivered the opposite ranking.

Table 2. Ranking of Alternatives $\tilde{a}_1$ and $\tilde{a}_2$.

Method	$\tilde{a}_1$	$\tilde{a}_2$	Ranking
E_j value under Weighted Method	0.52	0.87	$\tilde{a}_2 > \tilde{a}_1$
E_j value under Chang's Method	0.08	0.17	$\tilde{a}_2 > \tilde{a}_1$
Priority under Kaufmann and Gupta's Method	0.40	0.67	$\tilde{a}_2 > \tilde{a}_1$

Note: $x > y$ indicates that alternative x is preferred to y.

5. AN EXTENSIVE ILLUSTRATIVE EXAMPLE

Suppose that a firm must select between the two mutually exclusive alternatives A and B by using the IRR criterion. The estimated or most promising incomes and costs of these two alternatives are given in Table 3. Figure 2 illustrates the traditional or crisp representation of the net cash flows of these data.

Table 3. Costs C_t and Benefits B_t of Alternatives A and B.

	A		*B*	
t	C_t (\$)	I_t (\$)	C_t (\$)	I_t (\$)
0	7,000	0	11,0000	0
1	4,000	6,475	4,000	7,672
2	4,000	6,475	4,000	7,672
3	4,000	6,475	4,000	7,672
4	4,000	6,475	4,000	7,672
5	4,000	6,475	4,000	7,672

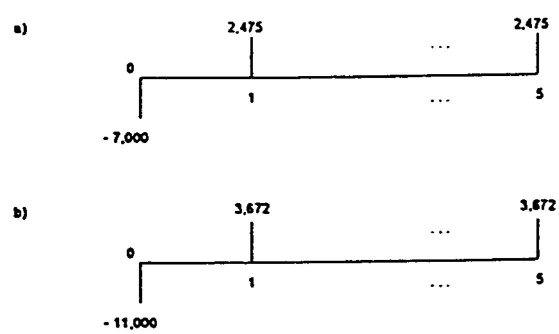

Figure 2. Crisp Net Cash Flows for (a) Alternative A and (b) for Alternative B.

By using the notation in (Pohjola and Turunen, 1990), the incomes and costs given in Table 3 can be represented as TFNs with most promising values m and with left and right spreads, $L = (1 - \pi_L\%)\times m$ and $R = (1 + \pi_R\%)\times m$, respectively. The parameters π_L and π_R represent the estimated level of vagueness to the left and to the right, respectively, of the most promising cash flow m. In Chiu-Yu *et al.* (1995), the equivalent parameters for π_L and π_R are measured in terms of *predicate units* (a preestablished unit of measure) to represent the level of confidence a decision maker has to the left and to the right of the most promising value m. Furthermore, they indicated that if the values of these parameters are either $\pi_L = \pi_R$, $\pi_L < \pi_R$, or $\pi_L > \pi_R$, then the estimation of the cash flows is said to be confident, confident to the right, or confident to the left, respectively, about the most promising value. Figure 3 shows the representation of the cash flows for the data in Table 3 when $\pi_L = \pi_R = \pm$ 5%.

For instance, for period $t = 0$, the TFN net cash flow of alternative A is $F_0 = (-7{,}000, -6{,}650, -7{,}350)$, and it is $F_0 = (-11{,}000, -10{,}450, -11{,}550)$ for alternative B. These two cash flows F_0 were obtained by using Eqs. (3), (4), and (5), respectively; the cash flows for the remaining periods were calculated in a similar way. It is important to point out here that although this example shows a symmetric confidence about the most promising value of the cash flows (i.e., $\pi_L = \pi_R$), asymmetry can be achieved by making $\pi_L \neq \pi_R$.

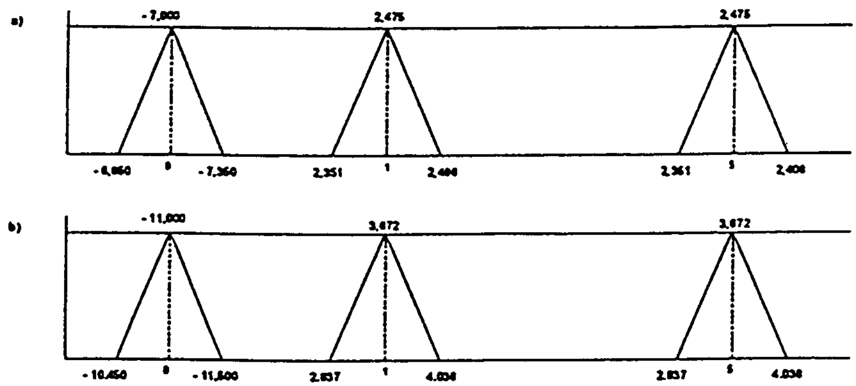

Figure 3. TFN Net Cash Flows for (a) Alternative A and (b) for Alternative B.

Next, when Eq. (6) is applied on the data in Figure 3, the IRRs in TFNs of the two alternatives are $i^*_A = (23\%, 10\%, 35\%)$ and $i^*_B = (20\%, 15\%, 30\%)$. Figure 4 shows the graphic representation of these two TFNs numbers. (It can be easily observed in Figure 4 that these two numbers are asymmetrical.) An inspection of this figure indicates that the TFNs i^*_A and i^*_B overlap, which suggests that it is not clear which of the two alternatives is the dominant one. In order to determine this dominance, the three methods described in Section 4 were used for ranking these two numbers. Table 4 shows the ranking of the two alternatives, and it also provides this ranking for the cases when $\pi_L = \pi_R = \pm$ 10% and $\pi_L = \pi_R = \pm$ 15%. (As it was mentioned previously, asymmetry on the cash flow of the alternatives can be achieved by making $\pi_L < \pi_R$ or $\pi_L > \pi_R$.).

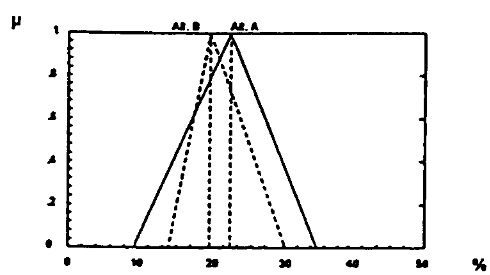

Figure 4. Fuzzy IRR values for alternative A and B (i.e., i^*_A and i^*_B, respectively) when $\pi_A = \pi_B = \pm$ 5%.

Table 4. Ranking of Alternatives A and B when $\pi_L = \pi_R$.

Method	$\pm\pi^{\dagger}$ (%)	Alt. A	Alt. B	Ranking§
i^* value (%) under Crisp method	5	23	20	A > B
E_j value under Weighted method	5	29	28	A >B
E_j value under Chang's method	5	0.028	0.015	A > B
Priority under Kaufmann and Gupta's method	5	0.226	0.211	A > B
i^* value (%) under Crisp method	10	23	20	A > B
E_j value under Weighted method	10	29	30	**A < B**
E_j value under Chang's method	10	0.057	0.033	**A < B**
Priority under Kaufmann and Gupta's method	10	0.2268	0.2262	A > B
i^* value (%) under Crisp method	15	23	20	A > B
E_j value under Weighted method	15	29	32	**A < B**
E_j value under Chang's method	15	0.088	0.056	A > B
Priority under Kaufmann and Gupta's method	15	0.226	0.242	**A < B**

§ $x > y$ indicates that x is preferred to y; $x < y$ reverses the preference.
† $\pi = \pi_L = \pi_L$.

The third and fourth columns of this table show the outcomes of the four ranking methods for the two alternatives. The ranking or dominance of the alternatives is summarized in the fifth column. From the data in the fifth column, it can be seen that under the crisp i^* criterion, alternative A was always preferred to alternative B. The reason for this dominance $A > B$ is obvious: The traditional IRR method computes the "i^* value with the most promising cash flows of the educated guesses" (Chui and Park, 1994). This conclusion can be easily verified by inspecting the values i^*_A and i^*_B for an $\alpha = 1.0$ in Figure 4. As indicated earlier, an *α-level* = 1 corresponds to the most promising value. This figure shows that at this *α-level* the relation $i^*_A > i^*_B$ is true, and hence the dominance $A > B$ is also true. It is also interesting to notice that the Alternative A also dominated alternative B (i.e., $A > B$) under the three "fuzzy" methods when $\pi_L = \pi_R = \pm 5\%$.

On the other hand, the data in the fifth column also indicate that when $\pi_L = \pi_R = \pm 10\%$ and $\pi_L = \pi_R = \pm 15\%$, then the ranking of both alternatives under the three "fuzzy" methods was inconsistent with the ranking of the traditional IRR. For instance, when $\pi_L = \pi_R = \pm 10\%$, the Weighted and Chang methods showed the ranking to $A < B$. Similarly, when $\pi_L = \pi_R = \pm 15\%$, the ranking with the Weighted and Kaufmann and Gupta's methods was also $A < B$. (This apparent nervousness among the various "fuzzy" ranking methods can be easily overcome by selecting only one method for the analysis, which according to Chui and Park (1994), such a method "must involve simple mathematical calculations.") The results of this example are interesting because they show that even small variations of the estimated cash flows can lead to a different recommendation, while the traditional IRR remains unchanged.

6. CONCLUDING REMARKS AND SUMMARY

The estimation of future cash flows for the selection of economic alternatives often relies on educated guesses either because the decision-makers do not possess enough knowledge, or because they do not have knowledge at all about the alternatives under consideration. Although these estimations are the result of human intuition, sophisticated mathematical modeling, or combinations of these, decision-makers often assign them a level of confidence (in terms of probability values). Therefore, the goal of decision-makers is to minimize the risk of making the wrong decision.

An accepted assumption during the selection of economic alternatives is that cash flows are normally distributed (Pike and Dobbins, 1986). In contrast, this paper presents a fuzzy theory approach in which cash flows do not rely on this assumption. Fuzzy theory allows decision-makers to model linguistic expressions as triangular fuzzy numbers (TFN) (Dubois and Prade, 1979). By using TFNs, decision-makers can express their educated guesses in terms of linguistic statements such as "an income of $500,000 plus or minus 10%."

Although the illustrative example presented here uses symmetrical TFN cash flows (i.e., the same confidence about the most promising value), it is quite simple to consider asymmetry to reflect estimations that are more realistic. The example presented here illustrated a methodology that can be easily extended for the utilization of asymmetric cash flows. By using asymmetric cash flows, the decision-makers can model their level of confidence to the left or to the right of the most promising value more realistically. More importantly, the utilization of asymmetric cash flows allows the decision maker to model cash flow estimation that with other methodologies would be hard to make.

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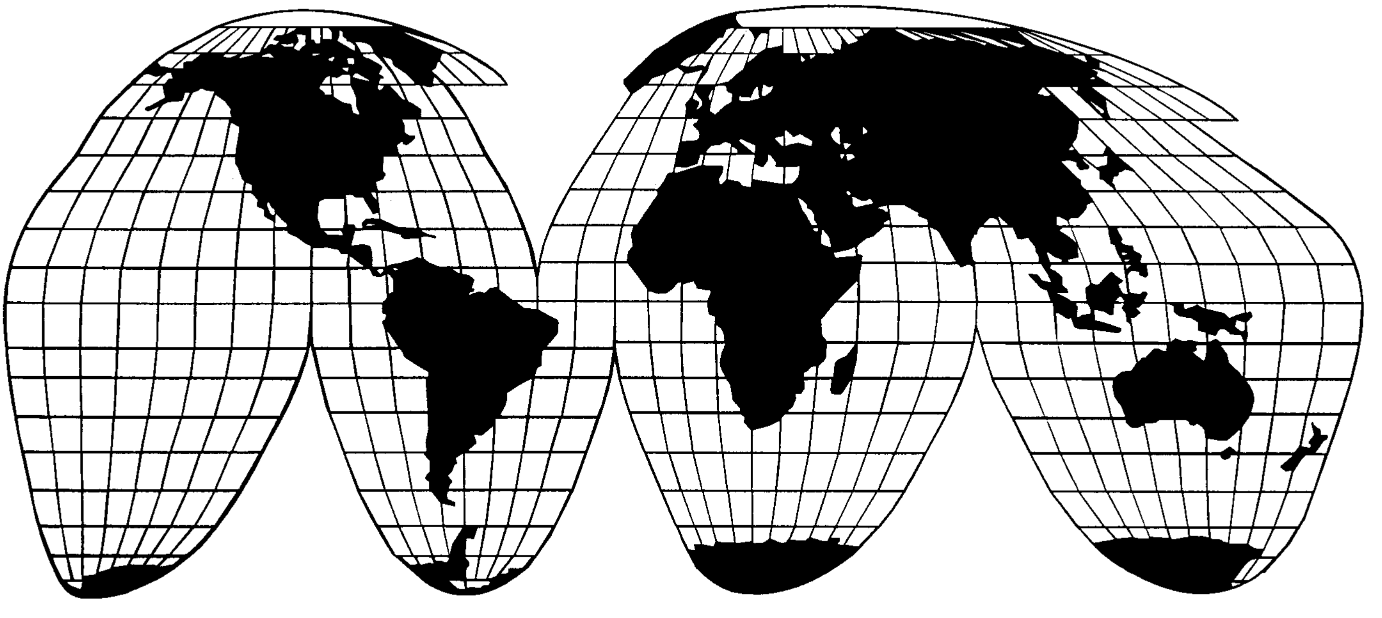

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