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Quantifying data for group technology with weighted fuzzy features

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The high potential of using group technology in manufacturing has attracted the interest of both practitioners and researchers. Group technology is based on clustering parts which have similar features. Very often it is very hard to quantify successfully data regarding these features. This is because in many real applications features are fuzzy. This paper identifies two types of fuzzy features: qualitative features, and quantitative ones with subjective meaning. The paper presents a methodology for quantifying the data that refer to the fuzzy features. The proposed methodology deals with crisp and fuzzy data in a unified manner. Finally, some clustering approaches which process the quantified features are also discussed.

1. Introduction

Contemporary group technology is becoming a mature technique of classifying parts into groups. The advantages of grouping parts are numerious and include increased productivity, reduced costs, reduced process planning effort, greater component standardization, improved cost estimation, reduced material handling, reduced setup times, better scheduling and faster product delivery.

Currently there are two main scientific approaches towards part classification: classification based on coding, and classification based on production flow analysis. The approach considered in this research is the coding approach for classification. More specifically, the problem discussed in this paper is grouping N different part types into groups based on a predetermined set of features.

Typical group technology methods classify the parts using codes which describe the parts' features. Such codes are standard and imply artificial dichotomy. No matter how the coding system is designed, there are always similar parts that are forced to have different codes.

Methodology presented in this paper identifies two main types of features: crisp features and fuzzy features. The crisp features are features that are uniquely and unambiguously defined by the feature's magnitude. Fuzzy features, on the other hand, have unclear or vague value. More specifically, we consider three types of features used for part grouping:

(1) Quantitative features. Such features represent properties of the parts that can be expressed numerically. They include a part's length, diameter, etc., and can be extracted from the CAD database, or measured directly if such a part exists.
(2) Qualitative (fuzzy) features. These features describe the part attributes in fuzzy terms such as 'large, medium, small', or other terms agreed upon by the system users. An example to such a feature is surface roughness which can be expressed as 'type A', 'type B', etc.

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(3) Quantitative features with subjective meaning. Features of this type have numerical values which do not quantitatively represent the actual meaning of these features in the relevant environment. Therefore, these features are also fuzzy. A feature of this type can be the length tolerance of a part. In a specific manufacturing environment a given tolerance may imply a different machining operation, or even an outside contract. Therefore, the interpretation of this feature type is specific to a particular design or manufacturing environment.

It is hard to underestimate the importance of the features of the second and third type in modern group technology applications, where traditional GT applications look only at features of the first type.

It is important to note that this paper is not intended to recommend what features to use for classification, but when such features are selected by a domain expert (designer, manufacturing engineer, etc.) the methodology presented defines the values these features have in the grouping process.

It is assumed that some of the features required for parts grouping are specific to the industry type and even the facility in use. This assumption is supported by the proliferation of coding systems that address diverse needs and users. Moreover, many classification systems allow the user to introduce his own information in order to incorporate the specific properties of the system into the classification algorithm.

This paper is structured as follows: section 2 presents a literature review of relevant group technology and clustering methods. Section 3 describes decision-making techniques based on fuzzy data. This background information is relevant since the present methodology is founded on the theory of decision making with fuzzy data.

Section 4 presents the methodology of analysing the three feature types previously described. The proposed methodology is demonstrated using an example throughout the paper. Section 5 demonstrates the clustering of the parts based on the features' values and weights (importance). Section 6 summarizes the methodology, and highlights future research needs.

2. Related work and background

2.1. *Group technology approaches*

Group technology (GT) is a manufacturing concept aimed at alleviating the production problems of a proliferation of products with decreasing life expectancy and improved quality. The GT methodology takes advantage of possible parts similarities. Parts can be grouped together based on design or manufacturing features (Groover 1987).

There are two main systematic approaches toward parts classification: parts coding, and production flow analysis (PFA). Parts coding is based on giving each part an individual code depending on its drawing or manufacturing features. Parts with similar codes belong to the same group. Part coding is a practical approach for various types of parts: mass produced parts; A B C part type analysis in a small quantity production (Opitz and Wiendahl 1971), electronic assembly (Styslinger and Melkanoff 1985), etc.

Part classification using PFA typically analyses a machines–parts binary matrix using a diversity of analytical techniques. Some of the matrix analysis methods used are single linkage cluster analysis (McAuley 1972), the bond energy method (McCormick *et al.* 1972), and the rank order clustering method (King 1980). Other approaches for

PFA grouping include integer programming (Kusiak 1987 b), a graph theoretical approach (Faber and Carter 1986), expert systems (Kusiak 1987 a), and heuristic methods (Askin and Subramanian 1987). PFA classification is based on rearranging the machine–parts matrix in such a way as to identify clusters of parts that share a group of machines. A more general approach is the analysis of a parts–properties matrix, in which the matrix can contain any numerical values.

However, in both contemporary group technology approaches part features used for the grouping are perfectly known, and discretized. Be it a part code, a routeing sheet, or a machines–parts matrix, all values must be perfectly known for the grouping process. Moreover, the properties by which the grouping is done are 'discretized' and therefore impose a dichotomy.

As an example, the first digit in the Opitz coding system (Opitz 1971) for rotational parts defines the length to diameter ratio. For ratios below 0·5 this digit has the value 0, while ratios from 0·5 to 3, gives the digit the value 1. Therefore, two parts with length to diameter ratios of 0·45 and 0·55 will be grouped based upon different properties (code digits in this example).

2.2. *Clustering procedures*

Techniques for cluster analysis seek to separate a set of data points into groups (clusters) of 'similar' points. Ideally there should be a unique clustering technique that would yield an obvious set of clusters for a given set of data. In practice, however, there have been a proliferation of clustering techniques, each using a different approach (for a review of clustering techniques see, for example, Everitt 1980).

Clustering techniques can be classified into five groups:

- Hierarchical techniques. In this case the classification is not performed in one step, but the classes themselves are classified into larger ones. This procedure forms a tree of clusters. Hierarchical techniques can be of two types: divisive or agglomerative. The divisive methods start with the set of N entities, and successively partition the set of finer and finer groups. Agglomerative methods work in the other direction.
- Optimization techniques. In this case the clusters are formed by the optimization of a clustering criterion.
- Density techniques form clusters by searching for regions containing a relatively dense concentration of points.
- Clumping techniques in which classes can overlap.
- Others.

All the above clustering techniques examine a set of parts in terms of a number of features. Information regarding these features is then used to cluster these parts. Since features might be quantitative, qualitative, or quantitative with subjective meaning, it is important to develop a clustering methodology which can deal with all these types of features in a unified manner.

In addition, it is necessary to transfer the data of these features into the same unit, otherwise, one may have to face a scaling problem. In this case features measured in one unit (e.g. kg) yields a different grouping than the same features expressed in a different unit (e.g. lbs).

In the proposed methodology data about different features are expressed in terms of membership values. That is, for each part we need to determine the membership value

of a given feature in the part. Usually, these values are numbers from the interval [0·00, 1·00]. The value 1·00 means that a feature is fully present in the current part, while a value of 0·00 means that the feature is not present at all. Values between 0·00 and 1·00 indicate different degrees of presence of the feature in the part. This method of data representation eliminates the scaling problem all together.

Since some features cannot be qualified precisely, the membership value problem here is similar to the membership value problem in fuzzy sets. For this reason the next section has a brief presentation of decision making with fuzzy data.

3. Decision making with fuzzy data

3.1. *Reciprocal matrices with pairwise comparisons*

Let $A_1, A_2, \ldots, A_n$ be the members of a fuzzy set. We are interested in evaluating the membership values (relative weights) of the above members. Saaty (1977, 1980) proposes use of matrix **A** of rational numbers taken from the finite set: $\{1/9, 1/8, \ldots, 1, 2, \ldots, 8, 9\}$. Each entry of the above matrix **A** represents a pairwise judgment. Specifically, the entry a_{ij} denotes the number that estimates the relative membership of element A_i when it is compared with element A_j. Obviously, $a_{ij} = 1/a_{ji}$ and $a_{ii} = 1$. That is, the matrix **A** is a reciprocal one. The pairwise comparisons are quantified by using the scale depicted in Table 1.

Besides the scale depicted in Table 1, other scales are possible as well. An evaluation of two families of scales can be found in Triantaphyllou *et al.* (1991). This idea of using pairwise comparisons has attracted the interest of many researchers (see for example, Federov *et al.* 1982, Khurgin and Polyakov 1986, Lootsma 1988, Triantaphyllou *et al.* 1990 a).

The main motivation of the pairwise approach is based on the fact that humans have serious difficulties evaluating many entities simultaneously. However, humans can perform rather well when they are asked to evaluate only two entities at a time. Therefore, it is highly desirable to develop a methodology that is based on data that can be derived from a sequence of pairwise comparisons.

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
3	Weak importance of one over the other	Experience and judgement slightly favour one activity over another
5	Essential or strong importance	Experience and judgement strongly favour one activity over another
7	Demonstrated importance	An activity is strongly favoured and its dominance is demonstrated in practice
9	Absolute importance	The evidence favouring one activity over another is of the highest possible order of affirmation
2, 4, 6, 8	Intermediate values between the two adjacent judgements	When compromise is needed

Table 1. Scale of relative importances.

Note that if object i has one of the above non-zero values when compared with object j, then j has the reciprocal value when compared with i.

Let us first examine the case in which it is possible to have perfect values for a_{ij}. In this case $a_{ij}=W_i/W_j$ (W_s denotes the actual value of element s) and the previous reciprocal matrix **A** is consistent. That is

$$a_{ij}=a_{ik}\cdot a_{kj} \qquad (i,j,k=1,2,3,\ldots,n) \tag{1}$$

It can be proved that **A** has rank 1 with $\lambda=n$ its non-zero eigenvalue (to be explained below). Then we have

$$\mathbf{A}x=n\mathbf{x} \qquad \text{where } x \text{ is an eigenvector} \tag{2}$$

From the fact that $a_{ij}=W_i/W_j$ the following are obtained:

$$\sum_{j=1}^{n} a_{ij}W_j=\sum_{j=1}^{n} W_i=nW_i \quad i=1,2,\ldots,n \tag{3}$$

or

$$\mathbf{AW}=n\mathbf{W} \tag{4}$$

Equation (4) states that n is an eigenvalue of **A** with **W** as the corresponding eigenvector. The same equation also states that in the perfectly consistent case (i.e. $a_{ij}=a_{ik}\cdot a_{kj}$) the vector **W** with the membership values of the elements $A_1, A_2,\ldots,A_n$, is the principal right-eigenvector (after normalization) of the matrix **A**.

3.2. *Eigenvalue approach*

In the non-consistent case (which is more common in practice) the pairwise comparisons are not perfect, that is, the entry a_{ij} might deviate from the real ratio W_i/W_j (i.e. from the ratio of the real membership values W_i and W_j). In this case, the previous expression (1) does not hold for all the possible combinations.

Now the new matrix **A** can be considered as a perturbation of the previous consistent case. When the entries a_{ij} change slightly then the eigenvalues change in a similar fashion (Saaty 1980). Moreover, the maximum eigenvalue is close to n (greater than n) while the remaining eigenvalues are close to zero. Thus, in order to find the membership values in the non-consistent cases, one should find an eigenvector that corresponds to the maximum eigenvalue $\lambda_{\max}$. That is to say, to find the principal right-eigenvector **W** that satisfies

$$\mathbf{AW}=\lambda_{\max}\mathbf{W}$$

where $\lambda_{\max}\approx n$.

One way to estimate the reciprocal right-eigenvector **W** is by multiplying the entries in each row of matrix **A** together and taking the nth root (n is the number of the elements in the fuzzy set). Since we desire to have values that add up to 1·0 we normalize the previously found vector by the sum of the above values. If we want to have the element with the highest value to have membership value equal to 1·0 we divide the previously found vector by the highest value.

The consistency of the judgements is not obvious and has to be examined. A measure of the consistency is the consistency ratio. The consistency ratio (CR) is obtained by first estimating $\lambda_{\max}$. Saaty estimates $\lambda_{\max}$ by adding the columns of matrix

A and then multiplying the resulting vector by the vector **W**. Then he uses what he calls the consistency index (CI) of the matrix **A**. He defined CI as follows:

$$\text{CI} = \frac{\lambda_{\max} - n}{n-1}$$

Then, the consistency ratio CR is obtained by dividing the CI by the random consistency index (RC) as given in the following table:

n	1	2	3	4	5	6	7	8	9
Random consistency index (RC)	0	0	0·58	0·9	1·12	1·24	1·32	1·41	1·45

Each RC is an average random consistency index derived from a sample of 500 randomly generated reciprocal matrices with entries from the set: $\{1/9, 1/8, \ldots, 1, 2, \ldots, 8, 9\}$. If the previous approach yields a CR greater than 0·10 then a re-examination of the pairwise judgements is recommended until a CR less than or equal to 0·10 is achieved. However, evaluations of the eigenvalue approach reveal that the CI coefficient needs to take very small values (almost 0·00) in order for the final results to be reliable (see for example Triantaphyllou and Mann 1990 and Triantaphyllou *et al.* 1990 b).

3.3. *Analytic hierarchy process*

Part of the analytic hierarchy process (AHP), (Saaty 1977, 1980), deals with the structure of an $N \times M$ matrix where N is the number of alternatives (e.g. part types), and M is the number of criteria (features in our case). The problem is to optimize the alternative selection decision. This matrix is constructed using the relative importances of the alternatives in terms of each criterion. The vector $\{a_{i1}, a_{i2}, \ldots, a_{iN}\}$ describes the impact of the ith criterion on each alternative. This vector is the principal eigenvector of an $N \times N$ reciprocal matrix which is determined by pairwise comparisons of the N alternatives with regards to each criterion: M such matrices are constructed.

Some evidence is presented in Saaty (1980) that supports this technique for eliciting numerical evaluations of qualitative phenomena from experts and decision-makers. The above $N \times N$ reciprocal matrices are processed as in the previous sections. That is, the a_{ij} values are determined by using the pairwise comparison approach. The entry a_{ij}, in the $N \times M$ matrix, represents the relative value of the alternative A_i when it is considered in terms of criterion j.

In AHP

$$\sum_{i}^{N} a_{ij} = 1$$

According to AHP the best alternative (in the maximization case) is indicated by the following relationship:

$$A_{\text{AHP}}^{*} = \text{Max}_i \left\{ \sum_{j=1}^{M} a_{ij} E_j \right\} \qquad \text{for } i = 1, 2, 3, \ldots, N$$

A variation of this method is further detailed in the following section.

3.4. *Revised analytic hierarchy process*

Belton and Gear (1983) proposed a revised version of the AHP model. They demonstrate that an inconsistency can occur when the AHP is used. According to the authors the root of that inconsistency is the fact that the relative values for each criterion sum up to one. Instead of having the relative values of the alternatives $A_1, A_2, \ldots, A_M$ sum up to one, they propose to divide each relative value by the maximum of the relative values.

In Triantaphyllou and Mann (1989) the AHP, revised AHP and two other multicriteria decision-making methods were evaluated. These analyses suggested that the revised AHP seems to be the best method from the ones examined. Therefore, the proposed GT methodology is based on the revised AHP method.

4. Deriving membership values

This section discusses the process of generating membership values for the selected features using a variation of the revised AHP method. This methodology however, does not require pairwise comparisons of all parts combination, which is impractical for a large collection of parts. Instead, the fuzzy features which have discrete number of values are compared. This comparison results in a $K \times K$ matrix for each feature with K discrete values. Each such matrix describes the degree of which a certain feature is present in the parts, therefore M such matrices are generated for M different features. Clearly, the number K of different feature values may vary in the different features.

Methodology of deriving the data required for grouping parts is demonstrated using an example. In this example there are seven parts which can be characterized in terms of four features. The features used in this example are: 'length', 'diameter', 'surface roughness', and 'central hole depth', as depicted in Table 2. The meaning of these features is implied by their names. From these four features the first two are considered to be quantitative. The 'surface roughness' feature is considered to be a qualitative one, while the 'central hold depth' is a quantitative feature with qualitative (subjective) meaning.

This example is used extensively in the rest of the paper, and the first step is the derivation of the membership values of the four features in the seven parts.

4.1. *Membership values for quantitative features*

This is the simplest case. Since the seven parts can be expressed precisely in terms of quantitative features, the pertinent membership values can be derived directly from the

Part type	Length	Diameter	Surface roughness	Centre hole depth
A	10·00	2·25	very high	7·00
B	8·50	4·00	high	0
C	5·50	5·00	medium	2·00
D	3·75	3·25	low	1·00
E	6·25	1·75	very low	0
F	8·00	2·50	high	4·00
G	7·50	3·00	very high	3·00

Table 2. Parts features matrix.

Part	Length	Diameter
A	1·000	0·450
B	0·850	0·800
C	0·550	1·000
D	0·375	0·650
E	0·625	0·350
F	0·800	0·500
G	0·750	0·600

Table 3. Membership values of length and diameter.

quantitative data. For the case of the 'length' feature, the data are represented by the vector [10·00, 8·50, 5·50, 3·75, 6·25, 8·00, 7·50]. The relative membership values of this feature can be derived by dividing each of the length values by the largest number in that vector (i.e. 10·00). That is, the pertinent membership values are: [1·000, 0·850, 0·550, 0·375, 0·625, 0·800, 0·750].

Similarly, the relative membership values for the 'diameter' feature are represented by the vector [0·450, 0·800, 1·000, 0·650, 0·350, 0·500, 0·600]. Membership values for the two quantitative features are described in Table 3.

However, when qualitative or quantitative with subjective meaning features are present, the situation is more complicated. The next subsection illustrates this situation.

4.2. *Membership values for qualitative features*

Consider the qualitative feature 'surface roughness'. The seven parts A, B, C, ..., G can be characterized in terms of this feature by using one of the following five descriptors (Table 2.): very high, high, medium, low, very low. The problem on hand is to quantify these five descriptors.

As was mentioned before, a method based on pairwise comparisons is applicable. These pairwise comparisons need to compare each of the previous five descriptors with each other. In other words, we need to answer questions like 'how much more important is *very high* when it is compared with *medium*?' By using the set of answers given in Table 1, this process leads to the construction of a reciprocal matrix with pairwise comparisons. Once this matrix is constructed, the eigenvalue approach presented in section 3 can be applied, and the previous five descriptors can be quantified.

Suppose that when the previous five descriptors are compared by using the set of answers depicted in table 1, the following matrix with pairwise comparisons is derived (Table 4).

For instance, the (1, 2) entry in the previous matrix is 3 because when the 'very high' descriptor was compared with 'high' the answer was assumed to be 'weak importance' which is assigned the numerical value 3 in Table 1. The interpretation of the remaining numerical values in the previous matrix is similar.

In order to generate the eigenvector for this matrix, the following computation is used (as mentioned in section 3.2):

$$V_i = \left(\prod_{j=1}^{N} a_{ij} \right)^{1/N} \tag{5}$$

	Very high	High	Medium	Low	Very low
Very high	1·00	3	5	7	9
High	0·333	1·00	3	5	7
Medium	0·200	0·333	1·00	3	5
Low	0·143	0·200	0·333	1·00	3
Very low	0·111	0·143	0·200	0·333	1·00

Table 4. Pairwise comparisons for the surface roughness feature.

Where V_i is the value pertinent to row i. For example $V_i = (1*3*5*7*9)^{1/5} = 3{\cdot}936$.

When the eigenvector approach is applied to the previous reciprocal matrix, the following vector of numerical values is derived:

Descriptor	Numerical value	Normalized value
Very high	3·936	1·000
High	2·036	0·517
Medium	1·000	0·254
Low	0·491	0·125
Very low	0·254	0·064

Note, that the previous numerical values were normalized by dividing them by the maximum entry in that vector. This is the reason why 'very high' is associated with the entry 1·00. Once the five descriptors have been quantified the seven parts can be expressed numerically in terms of the 'surface roughness' feature by simply replacing the qualitative descriptors by their corresponding numerical values. In other words, the corresponding membership values are as in the following vector.

Part	Surface roughness
A	1·000
B	0·517
C	0·254
D	0·125
E	0·064
F	0·517
G	1·000

4.2.1. *Consistency verification*

In order to accept the recommended feature values, the consistency of the judgements has to be verified. This is done by testing that the CR (consistency ratio) is less than 0·1. The CR is calculated as follows:

(1) The eigenvector is normalized such that its entries sum to 1. The resulting vector is

$$\mathbf{v} = [0{\cdot}5100,\ 0{\cdot}2638,\ 0{\cdot}1295,\ 0{\cdot}0636,\ 0{\cdot}0329]$$

(2) λ_{max} is estimated. This is done in the following way: add the columns of the comparison matrix **A** to form a new vector **w**. $\lambda_{max} = \mathbf{vw}$. In the example

$$\lambda_{max} = 5{\cdot}2429$$

(3) Calculate CI (consistency index) where

$$CI = \frac{\lambda_{max} - n}{n-1} = 0{\cdot}06073.$$

(4) Find the consistency ratio:

$$CR = \frac{CI}{RC} = 0{\cdot}054 < 0{\cdot}10$$

where RC = 1·12 from the table in section 3.2.

In this case the comparisons are consistent enough to allow further grouping of the parts based on these data points.

4.3. *Membership values for quantitative features with subjective meaning*

The last feature 'central hold depth' is a quantitative feature with subjective meaning. This case can also be treated similarly to a qualitative feature.

Observe that the six descriptors [7·00, 4·00, 3·00, 2·00, 1·00, 2·00] are quantitative but with some qualitative characteristics with respect to this feature. Therefore, it is necessary to fully quantify them as well. This can be accomplished, as in the case of the qualitative descriptors, by using pairwise comparisons in a similar process.

Suppose that when an analysis similar to the one performed on qualitative features is concluded, the associated numerical value of the six descriptors are as follows.

Descriptor	Numerical value
'7·00'	1·000
'4·00'	0·850
'3·00'	0·750
'2·00'	0·450
'1·00'	0·100
'0·00'	0·005

Therefore, the seven parts can be expressed quantitatively in terms of the fourth feature as follows:

Part	Central hole depth
A	1·000
B	0·005
C	0·450
D	0·100
E	0·005
F	0·850
G	0·750

When the membership values of all the four features are considered together, Table 5 is derived. This table represents the previously described $N \times M$ matrix.

Part type	Length	Diameter	Surface roughness	Centre hole depth
A	1·000	0·450	1·000	1·000
B	0·850	0·800	0·517	0·005
C	0·550	1·000	0·254	0·450
D	0·375	0·650	0·125	0·100
E	0·625	0·350	0·064	0·005
F	0·800	0·500	0·517	0·850
G	0·750	0·600	1·000	0·750

Table 5. Feature membership values.

Feature	Numerical weight
Length	0·250
Diameter	0·10
Surface roughness	0·60
Central hole depth	0·05

Table 6. Feature weights vector.

Part	Length (0·250)	Diameter (0·10)	Surface roughness (0·60)	Centre hole depth (0·05)
A	1·000	0·450	1·000	1·00
B	0·850	0·800	0·517	0·005
C	0·550	1·000	0·254	0·450
D	0·375	0·650	0·125	0·100
E	0·625	0·350	0·064	0·005
F	0·800	0·500	0·517	0·850
G	0·750	0·600	1·000	0·750

Table 7. Summary of feature values and weights.

4.4. *Finding the relative weight of each feature*

The parts to be grouped are judged by evaluating their similarities based on the given M features. However, the features are not of equal importance, and the clustering process needs to consider the relative importance of each feature. Finding the relative importance of each feature can also be done using pairwise comparisons among the features, therefore, yielding an $M \times M$ matrix.

Such a process requires $M(M-1)/2$ comparisons, similarly to the fuzzy features matrix construction. This process results in a matrix which represents the preferences of the user. Different users with diverse grouping needs (and preferences) may get different matrices. Suppose that a user compares all the four features of the current example, and that when the corresponding reciprocal matrix is processed, the eigenvector presented in Table 6 represents the weights of the four features. These values are normalized to sum to 1. When these weights are combined with the membership values depicted in Table 5 the decision-making problem is formed, as described in Table 7.

5. Clustering the parts

Once the features that are used for the part grouping are described in terms of their membership values, and their relative importance, it is possible to cluster the parts into groups.

This section demonstrates two ways to conduct the grouping process. One is to use the entire membership value matrix (Table 7). The other way is to summarize the features of each part into an aggregate measure. The clustering then uses these values. The rest of this section demonstrates both approaches using the seven-parts example.

5.1. *Matrix-based clustering*

By multiplying the membership values of the features by the features' weight (importance) a new matrix is generated. In this matrix each part is represented as an M dimensional point in Euclidean space (each feature is a different dimension).

Using this matrix grouping is accomplished using any clustering method, preferably the hierarchical agglomerative approach. By applying the 'average distance between groups' (Sokal and Michener 1958) the dendogram (Fig. 1) is derived.

The dendogram presented in Fig. 1 shows that for a threshold value of 0·75 the parts families formed are {A, G}, {B, C, D, E}, and {F}.

5.2. *Aggregate-value clustering*

A somewhat simpler approach is to represent each part i using one aggregate value v_i. This value is calculated as follows:

$$v_i = \sum_{j=1}^{M} w_j a_{ij}$$

In this case there are M features, w_j is the weight associated with feature j, and a_{ij} is the membership value of feature j in part i (matrix value of cell (i, j)), as previously discussed.

In the example the vector **V** is:

$$\mathbf{V} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} = \begin{bmatrix} 0{\cdot}945 \\ 0{\cdot}603 \\ 0{\cdot}412 \\ 0{\cdot}239 \\ 0{\cdot}230 \\ 0{\cdot}603 \\ 0{\cdot}885 \end{bmatrix}$$

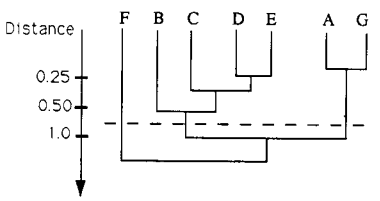

Figure 1. Dendrogram for matrix-based clustering.

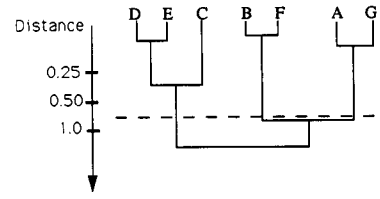

Figure 2. Clustering results using an aggregate feature.

Clustering the parts based on this single value constitutes the dendogram presented in Fig. 2. As before, the clustering method is agglomerative using average distance between groups.

Figure 2 shows that similar grouping results from this approach. The groups formed at the threshold value of 0·75 are {A, G}, {C, D, E}, and {B, F}.

5.3. *Comparison with non-fuzzy GT methods*

In order to shed more light on the approach presented here, a comparison to two other non-fuzzy GT methods is provided.

Opitz code. The seven parts presented in the example differ in their L/D ratio, therefore, they may have a different first digit in the code. All the other properties of the parts in the example do not affect the Opitz code, therefore the parts constitute the following two groups: {A, E, F}, {B, C, D, G}. These groups are very coarse; for example, group {A, E, F} contains parts with very high and very low surface roughness, and also parts with a large centre hole and a part with no hole at all.

Regular clustering. In this approach the same agglomerative method presented in section 5.1 is applied to the parts, considering the quantitative data presented in Table 2. Therefore, the surface roughness property is not considered at all in this approach. The groups resulting from a threshold value of 0·85 are: {A}, {C, D}, {B, E, F, G}. These results are closer to the results presented in section 5.1, but still this approach is less informative, and less precise than the one presented earlier.

6. Conclusions

This paper presents a methodology for quantifying part features for grouping. The features can be of three types: quantitative, qualitative, and quantitative with subjective meaning. In the latter case the numerical value of the feature does not correspond to its actual meaning in the given context. Both latter feature types are considered fuzzy.

Methodology presented is based on a modification to the revised analytical hierarchy process (revised AHP). In this methodology the discrete values of the fuzzy features are compared to each other. This allows the same feature to be used for classification of a large collection of parts, unlike the more traditional part to part comparison. For fuzzy features M such matrices are generated. Calculating the eigenvector for each such matrix generates a vector which expresses the feature value for each part.

The next step is to find the relative weight (importance) of each feature. This is also done by using pairwise comparisons. The last step is to cluster the parts based on the features' values and weights. The paper presents two approaches towards the clustering procedure.

Advantages of the methodology presented are several: first, it allows adoption of the group technology methodology to specific features and specific needs. It enables the user to define the features used for parts grouping, even if they are not strictly numerical. Also the user can define the importance of the features in the specific scenario of interest. In addition this methodology generates homogeneous data for the clustering, thus eliminating the scaling problem of the different measures. This method is not constrained by the number of parts to be grouped, but by the number of different values the features can have.

The main disadvantage of the proposed methodology arises from the need to estimate $K \times (K-1)/2$ pairwise comparisons (K is the number of the qualitative descriptors). When K increases, then the required number of comparisons increases very fast. However, for average sizes of K the number of required comparisons is rather moderate. Furthermore, the need to reach an acceptable CI (consistency index) value of 0·10 or smaller, may require re-evaluation of the comparisons during the process, thus resulting is a smaller number of comparisons generated.

Also, the difficulty in estimating a comparison of the type 'How much more important is very high when it is compared with medium', is not a severe limitation of this method. Simply, the proposed method attempts to capture the fuzziness and imprecision involved when one deals with qualitative features. It is very important to keep in mind that the proposed method asks the decision-maker to do his best in evaluating two items at a time.

There are several topics for further research in this area. Minimizing the number of features required to achieve a reliable grouping is one. Some features are either very important or very polarized and allow clustering based on a minimum number of features. This approach can save a large amount of data storage and processing used in group technology. In other words, the sensitivity of the final clustering to the features' weights is of great importance. A related study in the area of multi-attribute decision making (Sanchez and Triantaphyllou 1991) has revealed that sometimes criteria (or features in our case) with the highest weights may not be the most critical ones. Clearly, more research in this direction, from a group technology point of view, is needed.

Another question of interest is to optimize the grouping for a given objective. Such an objective can be fuzzy as well, for example assemblability of the groups. These topics expand to the group technology methodology to new areas such as grouping for assembly.

Hence, the proposed methodology is expected to expand the range of group technology applications to new areas such as feature-based design (and grouping), GT for assembly design and operation, etc.

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