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Procedures for the evaluation of conflicts in rankings of alternatives

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Abstract

A critical problem in decision analysis is how to evaluate the difference between two or more different rankings of a set of alternatives. In this paper it is assumed that a group of decision makers has already established a ranking for each one of a set of alternatives. Then, the problem examined here is how to evaluate the differences among these rankings. This problem does not have a unique solution and thus we consider a number of alternative approaches. The analysis presented here illustrates that this problem is not trivial and, moreover, it is associated with some counter-intuitive issues.

Keywords: Decision analysis; Ranking of alternatives; Combinatorics; Conflict resolution; Group decision making

1. Introduction

There is a plethora a methods for determining the ranking of a set of alternatives in terms of a set of decision criteria. The interested reader may want to check with the work reported in [1–7]. Some of the industrial engineering applications of decision theory include its use in integrated manufacturing [8], in the evaluation of technology investment decisions [9], in flexible manufacturing systems [10], layout design [11], and also in many other engineering problems (see, for instance, [12–21]).

In most of the engineering and business applications of decision theory there is a group of decision makers. Although there is a lot of work on the use of decision theory in

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engineering problems, there is very little work on how one can evaluate the conflict of having different rankings for the same set of alternatives. This is a problem which arises when there is more than one decision maker. That is, this problem is typical in group decision making [22–25].

The importance of group decision making (GDM) becomes evident by the realization that in most cases decisions are made by groups as opposed by a single decision maker (see, for instance, George *et al*. [26] and Hackman and Kaplan [27]). The need to improve group decision making has long been recognized by many researchers (e.g. DeSanctis and Gallupe [28] and Shaw [29]). For some more recent views in these issues, the interested reader may want to consult with the survey by Faure *et al*., [30]. Thus, the domain of applicability of this paper is GDM as opposed to a single decision maker environment.

There are many compelling reasons why one may be interested in evaluating the conflicts in different rankings. By better understanding and quantifying these conflicts, it is easier for decision makers to reach an agreement or consensus by minimizing the required compromise (see also [31–34]). It is also possible to identify subgroups of decision makers who have the same or similar opinions in some key issues.

The problem examined in this paper is best described as follows: suppose that a given set of n alternatives or concepts, denoted as $A_1, A_2, A_3, \ldots, A_n$, are ranked differently by a number of decision makers; How can these conflicting rankings be compared? The following definition will be used extensively in this paper:

Definition 1. A *ranking* is defined as an ordering on the set of alternatives.

For simplicity, we will use only the indexes to denote a specific ranking. For instance, if we deal with the four alternatives (or concepts) A_1, A_2, A_3, A_4, then one possible ranking is: (2, 3, 4, 1). That is, alternative A_2 is the most preferred alternative, alternative A_3 is the next most preferred one, etc.

Suppose that $[i_1, i_2, i_3, \ldots, i_n]$ and $[j_1, j_2, j_3, \ldots, j_n]$ are two possible rankings of n alternatives or concepts. That is, a ranking is an n-tuple of integers between *1* and n (with no two elements being identical). Then, the problem examined in this paper is how one can express the conflict or difference between these two rankings. The following section introduces the terminology used in this paper. After the pertinent terminology is introduced, the general problem is discussed in the third section. Some theoretical results are discussed in the fourth section. These results are in some sense counter-intuitive and useful in leading to a better understanding of the differences in different rankings. The last section discusses the main conclusions and findings of this paper.

2. Definitions and terminology

Besides the notion of ranking, which was introduced in the previous section, some additional key concepts are important in dealing with the conflict of rankings problem.

Definition 2. *Rank* is the position given to a specific alternative or concept within a given ranking. If there are n concepts to be ranked, then the rank is an integer number between *1* and n. Rank of *1* is assumed to be the highest, while rank of n is considered to be the lowest. Moreover, it is assumed that no two concepts can have the same rank.

Definition 3. *Agreement*: this state is a harmony of opinion that occurs when different decision makers conclude that a given concept is of a specified rank.

Definition 4. *Total agreement*: agreement upon each and every rank of the n concepts given for ordering.

Definition 5. *Conflict*: the situation existing when decision makers fail to agree upon two or more concepts.

3. The general problem context

An important issue in understanding this problem is first to decide what is the reason that one wants to rank a number of concepts. Once this issue is understood, then it is easier to decide which ranking difference evaluation method is more appropriate.

Generally, we are considering rankings of a set of ideas or concepts in some order, normally from the most important to the least important. For a 5-tuple of concepts designated by A_1, A_2, A_3, A_4 and A_5, one possible ranking would be (note that for simplicity we use only the indexes): (5, 1, 4, 2, 3). This ranking really has no significance singly other than to say that it is the way a specific individual ranked the importance of the set. When the same set is ranked by a second individual, the ranking might be: (3, 4, 1, 5, 2).

Next, consideration must be given to the fact that the two people have different opinions with regard to the importance of the concepts contained within the set $\{A_1, A_2, A_3, A_4, A_5\}$. These differences of opinions are termed as *conflicts* (definition 5, above). The goal of this paper is to develop a method of classifying or evaluating conflicts like the above ones.

In order to pursue the above challenge, consider the 5 member set mentioned previously: $\{A_1, A_2, A_3, A_4, A_5\}$. Assume for a basis of analysis that this order is the "reference" ranking. That is, the reference ranking is assumed to be: (1, 2, 3, 4, 5). This reference ranking may represent the ranking derived by a key decision maker. If one wishes to evaluate how different two given rankings are, then any one of these rankings can be considered as the reference ranking. This is explained more later in Section 6.

Let us first consider what the minimum deviation (or minimum conflict) would be from this reference ranking. Obviously, if the ideas are ranked in order of importance, the least important concepts will appear in the last two positions. Hence, an inversion of order in these two last positions would yield the minimum conflict situation in the ranking. Further, common sense would dictate that the maximum conflict would occur with a total inversion in order, i.e. when the ranking is: (5, 4, 3, 2, 1).

In order to evaluate a ranking it is necessary to convert the differences of the rankings into numerical data. One method of valuating the ranking conflict would be to evaluate a ranking by the sum of the squares of the differences of the assigned ranks given by the different persons completing the ranking. In applying this method to the previously discussed hypothetical illustrative example, the following is revealed:

Case I:	**Original Ranking:**	1 2 3 4 5
	Minimum (non-zero) Deviation:	1 2 3 5 4
	Sum-of-Squares:	$(1\text{-}1)^2 + (2\text{-}2)^2 + (3\text{-}3)^2 + (4\text{-}5)^2 + (5\text{-}4)^2 = 2$
Case II:	**Original Ranking:**	1 2 3 4 5
	Specific Ranking:	3 4 1 5 2
	Sum-of-Squares:	$(1\text{-}3)^2 + (2\text{-}4)^2 + (3\text{-}1)^2 + (4\text{-}5)^2 + (5\text{-}2)^2 = 22$
Case III:	**Original Ranking:**	1 2 3 4 5
	Maximum Deviation:	5 4 3 2 1
	Sum-of-Squares:	$(1\text{-}5)^2 + (2\text{-}4)^2 + (3\text{-}3)^2 + (4\text{-}2)^2 + (5\text{-}1)^2 = 40$

When the above conflict rankings are normalized by dividing each through by the maximum value (i.e., the value 40), the previous numerical results become: 0.05, 0.55 and 1.00, for case I, II and III, respectively. These values seem relevant as a measure of conflict since they associate a lower value with minimum conflict and the value 1.0 with total disagreement.

Obviously, one can think of more methods for evaluating the difference in conflicts in rankings. The purpose for which the ranking is to be made is in some cases a determinant of the method of evaluation to be used. The context of the ranking situation is also relevant. Diplomats might find, for example, that agreement on a single issue is of importance in the beginning of a negotiation. With that agreement, a common ground has been found upon which the continuance of the process can be based. In some cases the agreement on the subordinate issue will give grounds for progress toward agreement on an issue of primary importance. In the next section, we consider some possible ways for evaluating conflicts in rankings.

4. Some possible methods for the evaluation of conflicts in rankings

4.1. Number of disagreements

One of the most elementary ways of comparing the results of rankings is to count the number of disagreements. This method would provide good results in situations where there is a minimal difference between the rankings. That is, in cases in which the concepts or ideas considered are noncontroversial or not of major significance. For case III, above, the numerical value of the conflict (disagreements) under this method would be equal to 4. Obviously, a measure of the harmony between two rankings would be the number of agreements.

An obvious problem with this method is that disagreement in the first position of a ranking often might be more significant than disagreement in the last position of the ranking. This leads the analyst to the next procedure which is a logical extension of this method.

4.2. Weighted number of disagreements

If the ranking of a certain issue is of significant importance or it is controversial, the disagreements can be weighted according to the importance of the issues to be ranked. That is, this is a weighted version of the previous method. It is reasonable to conclude that in certain cases issues would not be of equal importance. Relative importance could be reflected in a weighting of each issue by a primary decision maker (i.e., the person who is seeking input from others.)

4.3. Sum-of-squares of differences in rank

This method provides for discrimination between major differences in ranks. Its minimum sum is zero when there is a total agreement in ranking. Its maximum will occur in cases of total inversion of order of ranking.

4.4. Normalized sum-of-squares of differences in rank

This procedure is identical to the above but a normalization occurs by dividing each sum by the maximum sum thereby forcing the range of values to fall in the interval [0, 1].

4.5. Sum of the absolute values of differences in rank

This procedure somewhat de-emphasizes the importance of major differences in ranks. It weights the difference in ranks the same regardless where the difference occurs. In the next section the above five conflict evaluation methods are analyzed and some intriguing observations are derived.

5. Analysis of the methods of evaluation of rankings

The main concept of the undertaken analysis is best illustrated via a simple example. Next we consider the case ranking three entities A_1, A_2 and A_3. There are 6 (i.e., $3! = 6$) possible rankings for this case. We assume that the reference ranking is: (1, 2, 3). Then, Table 1 depicts the conflict values for each one of the possible rankings when it is examined in terms of the previous five conflict evaluation procedures (termed as method 1, 2, 3, 4 and 5, respectively).

In Table 1 for the weighted number of disagreements method (or method 2), the weights which were assumed were equal to 3.0, 2.0 and 1.0, for the first, second and third position, respectively.

Table 2 is derived from Table 1 by observing that under method 1, there are at most 3 possible values, under method 2 there are 6 possible values, under method 3 there are 4 possible values, etc. That is, Table 2 depicts the rank preferences assigned by each one of the previous five methods.

Table 2 illustrates that all of the methods agree upon the first two rankings and with the exception of the weighted agreement method (i.e., method 2), the preferences are the same for the first five rankings. It appears that the weighted agreement method gives a better discrimination between rankings in this situation since it divides the six possibilities into five distinct categories and has a supporting logic which has some merit. This logic being that when

Table 1
Conflict values for all possible rankings for a set with three concepts (the reference ranking is: (1, 2, 3))

Current ranking	Conflict value under different evaluative procedures				
	method 1	method 2	method 3	method 4	method 5
(1, 2, 3)	0	0	0	0.00	0
(1, 3, 2)	2	3	2	0.25	2
(2, 1, 3)	2	5	2	0.25	2
(2, 3, 1)	3	7	6	0.75	4
(3, 1, 2)	3	9	6	0.75	4
(3, 2, 1)	2	8	8	1.00	4

there is a single agreement, then the rank of the item for which the agreement is upon is of importance.

The previous illustrative example exhaustively examines the case of ranking a set with three members, indicating that the five conflict evaluation methods result in similar conclusions.

Method 1, the *number of disagreements*, is a good composite indicator in cases where the factors ranked are considered to be of equal weight. In a situation where the number of alternatives is small, there will be little discrimination since disagreements must occur in pairs. It must be understood that this method is merely counting the number of positional agreements in the rankings.

Method 2, the *weighted number of disagreements*, is somewhat more discriminating than the first due to the weighting that is placed upon a positional agreement. It can be applied for cases when the primary decision maker can place weights upon the positions or when discrimination is desired to a higher degree in the evaluation process.

Method 3, the *sum-of-squares of differences in rank*, amplifies the differences when ranks are significantly different. It is a good discriminator for differences in that it "weights" differences by the difference. It differs from the weighted number of disagreements method in that the weighting is not positional.

Table 2
Rank preferences assigned by the five methods

Current ranking	Rank preference under different evaluative procedures				
	method 1	method 2	method 3	method 4	method 5
(1, 2, 3)	1	1	1	1	1
(1, 3, 2)	2	2	2	2	2
(2, 1, 3)	2	3	2	2	2
(2, 3, 1)	3	4	3	3	3
(3, 1, 2)	3	6	3	3	3
(3, 2, 1)	2	5	4	4	3

Method 4, the *normalized sum-of-squares*, is identical to method 3 other than the normalization process to reduce the magnitude of the numbers used in the ranking.

Method 5, the *sum of the absolute differences in rank*, does not amplify differences in ranking as does methods 2, 3 and 4. It fails to discriminate in the 3-tuple case between the rankings (2, 3, 1), (3, 1, 2) and (3, 2, 1). For the comparison basis (1, 2, 3) the second grouping indicates that alternative A_1 is better than alternative A_2 which is in agreement with the basic ranking. However, the ranking (3, 1, 2) has no positional agreement with the basic ranking while the inverted order has one positional agreement with the basic ranking. The question then becomes: Does the agreement upon one absolute position outweigh the agreement that one factor is more important than the other? The absolute difference method equates the two cases while the sum-of-squares methods places more weight upon the positional agreement.

6. An application

Suppose that a group of decision makers is interested in the ranking of the three alternatives A_1, A_2, and A_3. Since the number of alternatives is three, the total number of all possible rankings is 6 (= 3!). The decision makers feel that the most appropriate method for evaluating their differences is to use the third method (sum-of-squares of differences in rank) discussed in the previous section.

All the possible rankings are presented in Table 3. The same table also presents the relative differences between any pair of rankings when the third method is used. For instance, entry (2, 3) in this table represents the comparison between the rankings R_2 = (1, 3, 2) and R_3 = (2, 1, 3). Since the third method (sum-of-squares of differences in rank) is used to evaluate the differences between rankings, the difference between R_1 and R_2 is equal to 6. This is the value in cell (2, 3) in Table 3. The numbers within the parentheses denote the normalized values when the previous differences are divided by the maximum value (i.e., 8). Obviously, this table is a symmetric one.

An examination of the results in Table 3 reveals some interesting observations. If one is willing to accept that decision makers with rankings that differ less than 2 (or, equivalently, less than 0.25 when normalized) should belong to the same subgroup, then from Table 3 it can be derived that decision makers with rankings equal to R_1 or R_2 belong to the same subgroup.

Table 3
Differences between pairs of rankings for three concepts when the third method is used

Ranking pairs	R_1 (1, 2, 3)	R_2 (1, 3, 2)	R_3 (2, 1, 3)	R_4 (2, 3, 1)	R_5 (3, 1, 2)	R_6 (3, 2, 1)
R_1 (1, 2, 3)	0 (0.00)	2 (0.25)	2 (0.25)	6 (0.75)	6 (0.75)	8 (1.00)
R_2 (1, 3, 2)	2 (0.25)	0 (0.00)	6 (0.75)	2 (0.25)	2 (0.25)	6 (0.75)
R_3 (2, 1, 3)	2 (0.25)	6 (0.75)	0 (0.00)	8 (1.00)	2 (0.25)	6 (0.75)
R_4 (2, 3, 1)	6 (0.25)	2 (0.25)	8 (1.00)	0 (0.00)	6 (0.75)	2 (0.25)
R_5 (3, 1, 2)	6 (0.75)	2 (0.25)	2 (0.25)	6 (0.75)	0 (0.00)	2 (0.25)
R_6 (3, 2, 1)	8 (1.00)	6 (0.75)	6 (0.75)	2 (0.75)	2 (0.75)	0 (0.00)

Table 4
Subgroups of similar rankings according to different intervals with differences between rankings

Intervals with differences between rankings		
0.00 to 0.25	0.25 to 0.75	0.75 to 1.00
$\{R_1\}$	$\{R_1, R_2\}$	$\{R_1, R_2, R_3\}$
$\{R_2\}$	$\{R_1, R_3\}$	$\{R_1, R_3, R_5\}$
$\{R_3\}$	$\{R_2, R_4\}$	$\{R_1, R_4, R_5\}$
$\{R_4\}$	$\{R_3, R_5\}$	$\{R_2, R_4, R_6\}$
$\{R_5\}$	$\{R_4, R_6\}$	$\{R_3, R_5, R_6\}$
$\{R_6\}$	$\{R_5, R_6\}$	$\{R_5, R_3, R_6\}$

We denote this subgroup as $\{R_1, R_2\}$. Another similar subgroup can be formed by decision makers with rankings in the sets $\{R_1, R_2\}$, $\{R_1, R_3\}$, $\{R_2, R_4\}$, $\{R_3, R_5\}$, $\{R_4, R_6\}$ and $\{R_5, R_6\}$. Working as above for the case of having acceptable difference between rankings less or equal to 6 (or 0.75 when normalized), the subgroups with similar rankings are as follows: $\{R_1, R_3, R_5\}$, $\{R_1, R_2, R_3\}$, $\{R_2, R_4, R_6\}$, $\{R_1, R_4, R_5\}$, $\{R_3, R_5, R_6\}$ and $\{R_5, R_3, R_6\}$. Obviously, for differences less than or equal to 8 (or 1.00 when normalized) all the rankings belong virtually to the same group. These considerations are also summarized in Table 4.

The tabulation of the subgroups of similar rankings according to the acceptable maximum difference between any pair of rankings within each subgroup, offers a convenient mechanism for effectively and efficiently defining groups of decision makers with similar perspectives in the way the three alternatives A_1, A_2 and A_3 should be ranked. The number of decision makers can be any. If one wishes to study in a similar manner cases with more alternatives, then a similar approach should be followed.

It should be stated here, however, that the size of Tables 3 and 4 is equal to the total number of all possible rankings. That is, it is equal to the factorial of the number of the items to be ranked. Thus, this approach is practical for small to medium numbers of items to be ranked (say, up to 5). However, even larger sizes may still be considered, but the results will have to be computerized. Even so, the size will still be limited.

All the previous analyses are particular to the method used to evaluate the differences between rankings. In this application, we used the third method, but any one of the proposed five methods could be used instead.

7. Conclusions

The comparison of conflicts in rankings is an area for much study and for profitable application. It has much promise for applications in such diverse areas as negotiations, corporate executive succession, and selection of "like minded" or contrawise, diverse groups. It has promise for developmental ideas to this point undefined because of its simplicity of application.

Some other related work can be found in [35, 36]. In [35] some analytical formulas are developed for calculating the number of agreements for rankings of different sizes. In [36] it is shown that for a given problem (that is, when the number of alternatives m and the number of decision criteria n are given), then the number of all possible rankings is not equal to $m!$ as one may have expected, but it is a function of m and n and it can be significantly less than $m!$ Clearly, more research is needed in this important area of decision analysis.

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