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A study of the total inventory cost as a function of the reorder interval of some lot-sizing techniques used in material requirements planning systems

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Abstract

This paper compares the total inventory costs (TIC) of five lot-sizing techniques. The add-drop heuristic (ADH) is a capacitated technique and the lot-for-lot (L4L), fixed period quantity (FPQ), least unit cost (LUC) and the silver-meal heuristic (SMH) are uncapacitated techniques. The TIC is considered as a function of the reorder interval (RI). This comparison is based on the assumption that if both capacitated and uncapacitated techniques produce identical RIs, then their TICs must also be identical (although uncapacitated techniques do not reflect this fact). Empirical results suggest that the ADH technique yields considerably better (i.e. lower) TICs when the demand levels and the number of items are low. On the other hand, these results suggest that for high demand levels, the TICs of the four popular lot-sizing techniques are close to the near optimal cost obtained by the (most time-consuming) ADH technique. Some theoretical results on the performance of uncapacitated techniques are also presented. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Total inventory cost; Lot-sizing techniques; Reorder interval; MRP lot-sizing techniques

1. Introduction

This paper examines the total inventory cost (TIC) as a function of the reorder interval (RI) of five lot-sizing techniques used in material requirements planning (MRP) systems. Its main purpose is to compare the TIC of a new and near optimal lot-sizing technique called the add-drop heuristic (ADH) (Hill, Raturi & Sum, 1988; Hill & Raturi, 1992) with the TIC of four popular techniques (Haddock & Hubick, 1989). These popular techniques are the lot-for-lot (L4L), fixed period quantity (FPQ), least unit cost (LUC)

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Nomenclature

N	Number of items
M	Number of parent-items
B	Number of periods in the planning horizon
b	A period in B such that $1 \le b \le B$
i	Item number ($i = 1, \ldots, N$),
j	Parent number ($j = 1, \ldots, M$),
d_i	Total demand for item i
d_{ij}	Demand of item i due to parent j
H_i	Holding inventory cost of item i per unit per period
K_i	Setup cost for item i
$p(i)$	Set of parent-items for item i
$r(i)$	Set of work centers in the routing of item i
w	Work center or machine w for $w = 1, \ldots W$
m_i	Exponent of the reordering policy
n_i	Reorder interval of child-item i
$q_w(n)$	Average time in the queue, (as computed in Yao, 1985)
S_{iw}	Setup cost of item i at machine w
st_{iw}	Setup time of item i at machine w
t_{iw}	Run time for item i at machine w
C_w	The capacity of work center or machine w
L_w	Average load for work center w
P_w	Penalty cost for capacity violations. All P_w values were set equal to 10^{10} for $w = 1, \ldots, W$, as in Hill and Raturi (1992))

and the silver-meal heuristic (SMH). MRP lot-sizing techniques are inventory control tools that allow users to make decisions on the timing and quantities of inventory parts (Orliky, 1975). Therefore, knowing the performance of these techniques is very important in manufacturing control environments because it allows a user to identify the conditions under which new techniques, such as the ADH, are more advantageous over other techniques.

Traditionally, MRP lot-sizing techniques are benchmarked by comparing the TICs and computational times they produce to solve a given inventory problem (Berry, 1972; Lee, Ristroph, Zhu, & Ruangdet, 1983). Often, the TIC is defined by the sum of the cost of placing orders (i.e. the setup cost) plus the cost of carrying unused inventory from one period to subsequent periods (i.e. the holding cost) (Solomon, 1991).

When lot-sizing techniques only use the setup and holding costs, they are referred to as *uncapacitated* techniques because they do not include any type of constraints in the available resources (i.e. production capacity is unlimited) (Solomon, 1991). The L4L, FPQ, LUC, and SMH are examples of uncapacitated techniques. On the other hand, lot-sizing techniques are referred to as *capacitated* techniques when in addition to the setup and holding costs, the cost structure also considers some constraints on the available resources (i.e. the capacity is finite) (Solomon, 1991). The ADH technique is a capacitated technique, which satisfies a production capacity constraint in each period of the planning horizon.

A methodological problem exists when one wishes to compare the TIC of these five lot-sizing techniques because they belong to two different types of approaches and hence serve two different purposes. That is, a direct comparison of these techniques based on the TIC is not possible and thus a common ground must be found if these techniques are to be compared. It can be observed that the RI (i.e. a time performance measure) is a common performance measure that can be used to compare the five lot-sizing techniques mentioned above. The relevance of RI as a performance measure is illustrated by considering the following hypothetical scenario. Let $\mathrm{RI}_u(A)$ and $\mathrm{RI}_c(A)$ be the RIs of item A when it is computed under uncapacitated and capacitated approaches, respectively. Also, let $\mathrm{TIC}_u(A)$ and $\mathrm{TIC}_c(A)$ be the total inventory costs associated with these two RIs, respectively. It can be observed that if $\mathrm{RI}_u(A) = \mathrm{RI}_c(A)$, then it can be assumed that both approaches perform identically, and therefore that the corresponding total costs must also be equal. That is, $\mathrm{TIC}_u(A) = \mathrm{TIC}_c(A)$. The goal of this paper is to analyze the performance of the five lot-sizing techniques mentioned above by comparing the $\mathrm{TIC}_u(A)$ and $\mathrm{TIC}_c(A)$ values as functions of the RI.

The empirical results reported in this study suggest that the ADH technique delivered considerably better TIC solutions (i.e. lower costs) for low demand levels and for few numbers of items. However, these results also suggest that for high demand levels, uncapacitated techniques delivered TIC costs that were very close to the near optimal cost obtained by the ADH technique.

This paper is organized as follows. The notations used in this paper are given in the nomenclature section. Section 2 introduces the derivations of both the TIC and RI values of the L4L, FPQ, LUC, SMH and ADH techniques. Section 3 states the assumptions needed in order to perform the comparisons of the five lot-sizing techniques. Section 4 presents a numerical example of the computation of the TIC by using one iteration of the ADH heuristic. Then, Section 5 presents the computational results on the TICs. Finally, this paper ends with some concluding remarks and a summary section.

2. Formulation of the TIC of the five lot-sizing techniques

According to Solomon (1991), the cost structure of uncapacitated techniques is defined by the sum of the setup and holding costs. Hence, an initial formulation of this TIC is as follows:

$$\mathrm{TIC} = \text{setup cost} + \text{holding cost}. \tag{1}$$

When the notation described above is used, then the TIC of item i is stated as follows:

$$\mathrm{TIC}(i) = K_i + H_i. \tag{2}$$

The next four subsections present the TIC of the four uncapacitated techniques which are derived from Eq. (2).

2.1. The lot-for-lot technique

Under the L4L technique, orders for item i are placed in every period b (for $b = 1, 2, 3, \ldots, B$) (see, for example, Orliky (1975). Consequently, under this technique there is no holding cost, and thus the average cost per period (denoted by $\overline{\mathrm{TIC(l4l)}}$) is solely determined by the setup cost (K_b) for that period. That is, the following condition is true:

$$\overline{\mathrm{TIC(l4l)}} = K_b, \qquad \text{for } b = 1, 2, 3, \ldots, B. \tag{3}$$

2.2. The silver-meal heuristic

The SMH is an uncapacitated lot-sizing technique that minimizes the average inventory cost per period of item i. Under this technique, the entire planning horizon (i.e. the entire number of periods) is covered by a sequence of time windows. Orders for item i are placed at the beginning of each window. The size of this window is determined according to the following heuristic approach (see for example, Buffa, 1983; Hendrick & Moore, 1985; Schonberger & Knod, 1986; Taha, 1992). Let $\mathrm{TIC}_{1,t}$ (for $1 \le t \le B$) denote the cost of the first t periods (i.e. the first time window) under the assumption that the order size at the beginning of period 1 is the sum of demands $d_1, d_2, d_3, \ldots, d_t$. Therefore, it can be easily shown that the $\mathrm{TIC}_{1,t}$ for the first window is given by the formula:

$$\mathrm{TIC}_{1,t} = K_1 + \sum_{p=1}^{t-1} H_p \sum_{q=p+1}^{t} d_q. \tag{4}$$

Furthermore, the average cost per period in the first window, denoted as $\overline{\mathrm{TIC(smh)}_{1,t}}$, is given by Eq. (5) (Taha, 1992):

$$\overline{\mathrm{TIC(smh)}_{1,t}} = \mathrm{TC}_{1,t}/t. \tag{5}$$

Next, the length of the first window (i.e. the RI value) is determined by first calculating the average cost $\overline{\mathrm{TIC(smh)}_{1,t}}$, for $t = 1, 2, 3, \ldots$ and stopping when a cost $\overline{\mathrm{TIC(smh)}_{1,v}}$ is found such that $\overline{\mathrm{TIC(smh)}_{1,v}} < \overline{\mathrm{TIC(smh)}_{1,v+1}}$, or if $v = B$. The second window is determined in a similar manner by starting at the next period (or next time-bucket) after the end of the first window. This process of determining windows continues until the entire number of periods is covered by a sequence of windows.

2.3. The least unit cost technique

The LUC technique is another uncapacitated technique that minimizes the average cost per unit in each period of the planning horizon. Under the LUC, the TIC for the first time window is computed as in Eq. (4). Therefore, the average cost per unit for the first period in the window (denoted by $\overline{\mathrm{TIC(luc)}_{1,t}}$) is given by Eq. (6) (Taha, 1992):

$$\overline{\mathrm{TIC(luc)}} = \mathrm{TC}_{1,t} \Big/ \sum_{p=1}^{t} d_p. \tag{6}$$

Next, the size of the first window (i.e. the RI value) is determined by first calculating the cost $\overline{\mathrm{TIC(luc)}_{1,t}}$ for $t = 1, 2, 3, \ldots$ and stopping when a cost $\overline{\mathrm{TIC(luc)}_{1,v}}$ is found such that $\overline{\mathrm{TIC(luc)}_{1,v}} < \overline{\mathrm{TIC(luc)}_{1,v-1}}$, or if $v = B$. The subsequent windows are determined in a similar manner by starting at the period the previous window ended plus one.

2.4. The fixed period quantity technique

The FPQ is also an uncapacitated technique that reorders inventory at periodic intervals. Under this technique, the entire planning horizon is divided into several, predetermined equally sized windows. In this paper, the windows sizes are predetermined by using the two-step heuristic described in Pantunsinchai (1983) as follows. First, the demands of item i in all periods of the planning horizon is added and an average demand $\bar{D}$ is computed. Next, by using the demand $\bar{D}$, an optimal lot-size quantity (Q^*) is computed by using the EOQ formula. The setup and holding costs are assumed constant for all periods in the planning horizon. Then, the size of each window (i.e. the RI) is computed as the integer value of the ratio:

$$\bar{D}/Q^*. \tag{7}$$

If this ratio is less than one, the windows sizes are set to one period (Moily & Matthews, 1987).

2.5. The ADH heuristic

On the other hand, the ADH heuristic is a capacitated lot-sizing technique that minimizes the TIC per period by satisfying a power of two policy. That is, the length of the RI is determined by the expression 2^x (where x is ≥ 0 and an integer). According to Hill et al. (1988) and Hill and Raturi (1992), an initial formulation of this problem is as follows:

$$\begin{aligned}\text{Min TIC} = {} & \text{setup cost} + \text{holding cost} + \text{work in process inventory cost} \\ & + \text{queue inventory carrying cost}.\end{aligned} \tag{8}$$

Subject to:

$$n_i = 2^{m_i},$$

where $m_i \geq 0$ and an integer.When the previous notation is used, the above formulation becomes:

$$\text{Min TIC} = \sum_{i=1}^{N} \left\{ \sum_{w \in r(i)} S_{iw}/n_i + H_i \sum_{j \in p(i)} d_{ij}|n_i - n_j|/2 + H_i d_i \sum_{w \in r(i)} (st_{iw} + d_i n_i t_{iw}) + H_i d_i \sum_{w \in r(i)} q_w(n) \right\}. \tag{9}$$

Subject to:

$$n_i = 2^{m_i},$$

where $m_i \geq 0$ and an integer.Then, when a capacity violation occurs at work center w (for $w = 1, 2, 3, \ldots, W$) a penalty cost, P_w, is added to Eq. (9), which then becomes Eq. (10) (Hill et al., 1988):

$$\text{Min TIC}' = \left(\text{TIC} + \sum_{w=1}^{W} P_w \times \max(L_w - C_w, 0) \right). \tag{10}$$

Subject to:

$$n_i = 2^{m_i},$$

where $m_i \geq 0$ and an integer.The term L_w in Eq. (10) represents the average load at work center w. It is defined as the sum of the average setup time per period and the run times per period at work center w (for $w = 1, \ldots, W$). That is

$$L_w = \sum_{i \in w(j)} (st_{ij}/2^{m_i} + 2^{m_i} d_i t_{ij}). \tag{11}$$

Step 1. **compute** TIC' for $m_i \leftarrow 0$
$i \leftarrow 1$
$\delta m_i \leftarrow 1$
Step 2. **if** $\delta TIC(\delta m_i) < 0$, **then** $L_k \leftarrow L_k + st_{ik}/2^{(m_i+\delta m_i)} - st_{ik}/2^{m_i}, \quad k = 1,\ldots,K$
$TIC' \leftarrow TIC' + \delta TIC(\delta m_i)$
$m_i \leftarrow m_i + \delta m_i$
Step 3. $i \leftarrow i + 1$
Step 4. **if** $i \leq N$, **then** go to Step 2.
Repeat Steps 1 to 4 until no improvement is found.
Note that $\delta TC(\delta m_i)$ is defined as the change in TIC' due to change δm_i in m_i.

Fig. 1. The four-step algorithm to solve the NLP problems in Eqs. (9) and (10).

At this point, it can be observed that if n_i is replaced by 2^{m_i} in relations (9) and (10), then the formulation in these two relations becomes a nonlinear minimization problem (NLP), which the ADH technique attempts to solve by using the following four-step algorithm shown in Fig. 1 (as described in Hill et al., 1988).

3. Assumptions for comparing the uncapacitated and ADH techniques

The previous Eqs. (1) and (8) illustrate the difference between the cost structures of the four uncapacitated and the capacitated lot-sizing techniques. It can be observed that Eq. (1) indicates that while uncapacitated techniques require only the setup and holding costs to make their reordering recommendation, Eq. (8) uses four costs to compute the TIC. Furthermore, it can be seen that when a capacity violation occurs, then Eq. (8) becomes Eq. (10), which includes a fifth cost to penalize this situation. At this moment, it is clear that a methodological problem would exist if the four uncapacitated and the ADH lot-sizing techniques are compared based solely on TICs.

It is quite reasonable to propose the RI as a common performance measure in comparing capacitated and uncapacitated techniques. This is correct because the ADH is a lot-sizing technique which first computes the RI (called a power of two RI) to find a near optimal TIC. Moreover, uncapacitated techniques are often used in order to determine reordering policies of inventory items (Buffa, 1983; Hendrick & Moore, 1985; Taha, 1992). Furthermore, for a moment suppose that both capacitated and uncapacitated techniques produce identical RIs for a given inventory problem. In this case, it would be quite natural to suppose that their TICs are also the same. However, it should be noted that even to assert that the TICs of both approaches are identical, a common cost structure is needed if both techniques are to be compared. In the remaining of this paper, the *unconstrained* cost structure of the ADH technique (i.e. unconstrained Eq. (10)) is considered as this common cost structure mainly because it is the most comprehensive one. Therefore, in order to make the uncapacitated techniques *comparable* with the ADH heuristic, the following three steps are proposed (Nieto, 1995):

1. First, the RI of each inventory item will be computed by using an uncapacitated technique.
2. Then, *equivalent capacitated costs* (i.e. setup cost, holding cost, work in process cost, cost of inventory in queues, and a penalty cost) will be computed by applying a *single iteration* of the algorithm illustrated in Fig. 1 on the RIs computed in step 1.

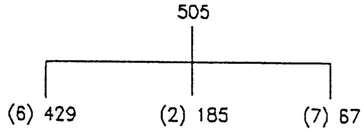

Fig. 2. Bill of materials of make-item #505.

3. Finally, the set of costs computed in step (2) will be added up in order to determine the uncapacitated lot-sizing technique TIC value.

Steps 1–3 will be repeated until all four uncapacitated techniques are used.

4. A numerical example

This section illustrates the computation of the equivalent capacitated costs for make-item #505 under the L4L technique. (The computation for other uncapacitated techniques were omitted here because of space considerations.) Fig. 2 illustrates the bill of materials (BOM) of item #505.

Table 1 shows the demand of the items in the BOM after item #505 was *MRP-exploded*. It also shows the holding cost, setup cost, and routing code of each item. The data in this table were arbitrarily assigned, but following the methodology described in Hill et al. (1988).

Table 2 shows a sequence of operations, setup times, and run times which are associated with each routing code. For example, routing code 1 indicates that operations 1 and 2 are performed in this routing. That is, the routing code of item #67 (Table 1) indicates that this item requires these two operations. Similarly, Table 3 defines the number of machines available at each operation. For instance, operation 2 can be performed with up to three machines, each available 8 hours per shift. For illustrative purposes, the data in Tables 2 and 3 were randomly assigned.

Table 4 summarizes the equivalent costs of the items shown in Fig. 2 when the RI n_i is one period long. The second column shows these RIs. The footnotes show details of the computation for the costs of item #505 after one iteration of the algorithm in Fig. 1 was executed. Similar computations were performed for the other items.

Table 1
Demand, holding and setup cost, and routing code

Item #	Demand[a]	Holding cost[b]	Setup cost[c]	Routing code
505	10	0.278	4.726	3
429	60	0.039	1.209	2
185	20	0.008	0.112	2
67	70	0.004	0.040	1

[a] Units.
[b] US $/unit/period.
[c] US $/order.

Table 2
Routing operation data (note: all times are in hours per period)

Routing code	Sequence of operations	Setup time	Run time
1	1	0.523	0.014
	2	0.523	0.033
2	3	0.216	0.010
3	2	0.120	0.002
	3	0.120	0.002

5. Computational results

This section describes the experimental conditions under which the five lot-sizing techniques were tested and the empirical results on the TICs. Results on CPU times were omitted because it was known beforehand that the ADH would take longer (actually exponential) CPU times than the uncapacitated techniques.

5.1. Experimental conditions

One thousand items were generated for this experiment. The first 500 items were designated as buy-items, whereas the remaining items were designated as make-items. Each item was associated with a holding cost, a setup-to-holding cost ratio, and a routing code, as follows. First, the holding costs of buy-items were randomly taken from $U[0.10, 60]$, while for the make-items they were computed by adding the holding costs of all the corresponding child-items (Hill & Raturi, 1992). For example, make-item #505 in Fig. 2 is composed of buy-items numbered #429, #187, and #67 (with holding costs of US$ 6, US$ 4, and US$ 2, respectively).Therefore, its holding cost is US$ 12 (= 6 + 4 + 2). Second, the setup-to-holding cost ratio of all thousand items was randomly taken from $U[5,35]$, following the experimental conditions in Hill and Raturi (1992). Finally, the routing code was randomly taken from $U[1,13]$; the number of available machines at each operation was assigned randomly from $U[1,10]$, also following the experimental conditions in Hill and Raturi (1992).

A BOM with 2400 relations of make-items and child-items was randomly generated. Each relation consisted of a make-item, a child-item, and a required quantity. The assignment of each make-item and child-item was made arbitrarily, but the quantity required by each make-item from its child-items was taken from $U[1,10]$, in accordance to the test conditions in Hill and Raturi (1992). For example, the relations {505, 429, 6}, {505, 185, 2}, and {505, 67, 7} (see also Fig. 2) compose the make-item #505 with child-items #429, #185, and #67, each with required quantities of 6, 2, and 7 units, respectively.

Table 3
Machine availability (note: as extracted from Hill and Raturi (1992))

Operation	Machines available	Hours per shift
1	1	8
2	3	8
3	1	8

Table 4
Equivalent costs (in US$) of make-item #505

Item # *I*	n_i	Setup cost[a]	Holding cost[b]	WIP cost[c]	Queue cost[d]	Total[e] cost/day
505	1	9.45	0.00	0.11	136.91	146.47
429	1	1.21	0.00	1.40	0.00	2.61
185	1	0.11	0.00	0.03	0.00	0.14
67	1	0.08	0.00	1.57	13.74	15.39
Totals:		10.85	0.00	3.11	150.65	164.61
Penalty Cost[f]:						0.00
TIC:						164.61
Operation *k*	Machine[d] utilization		Mean queue time[d]		Average load at *w*[d]	
1	1.500		0.00		1.50	
2	0.991		49.25		2.97	
3	1.370		0.00		1.37	

[a] The setup cost of item #505 in operations {2, 3} is as follows: $(S_{505.2} + S_{505.3})/n_{505} = (4.726 + 4.726)/1 = 9.452$.

[b] The holding cost for item #505 is computed by: $H_{505}d_{505.0}|n_{505} - n_0|/2 = 0.278 \times 10 \times |1 - 1|/2 = 0$. Observe that item 505 is assumed to have a 'fictitious' parent-item 0 with $n_0 = 1$. This is a common assumption to facilitate the manipulation of end-items.

[c] The cost of work in process inventory of item #505 in operations {2, 3} is determined by: $H_{505}d_{505}(d_{505}n_{505}t_{505.2} + d_{505}n_{505}t_{505.3}) = 0.278 \times (10)^2 \times (1 \times 0.002 + 1 \times 0.002) = 0.112$.

[d] The cost of holding inventory in queues for item #505 in operations {2, 3} is calculated by: $H_{505}d_{505}(q_2 + q_3) = 0.278 \times 10 \times (49.25 + 0) = 136.91$. The mean queue q_k value was taken from the lower part of Table 4. The M/G/c (Yao, 1985) model was used to estimate this q_k. This model was also used in Hill and Raturi (1992).

[e] The total inventory cost per period of item #505 is the summation of the individual cost for each item. It is TC = 9.45 + 0.00 + 0.11 + 136.91 = 146.47.

[f] The average load, L_k, of machine k due to the routing of item #505 is determined by: $L_1 = 0.0$, $L_2 = st_{505.2}/n_{505} + t_{505.2} \times D_{505} \times n_{505} = 0.12/1 + 0.002 \times 10 \times 1 = 0.14$ h, and $L_3 = st_{505.3}/n_{505} + t_{505.3} \times D_{505} \times n_{505} = 0.12/1 + 0.002 \times 10 \times 1 = 0.14$ h, (notice that L_1 is zero because machine 1 is not in the sequence of operations of the item.) The average loads, L_k, due to all four items are $L_1 = 1.505$, $L_2 = 2.973$, and $L_3 = 1.372$. The penalty cost, for capacity violations at all machines k, is computed as: $10^{10} \times \mathrm{MAX}(L_1 - c_1, 0) + 10^{10} \times \mathrm{MAX}(L_2 - c_2, 0) + 10^{10} \times \mathrm{MAX}(L_3 - c_3, 0) = 10^{10} \times [\mathrm{MAX}(1.503 - 8, 0) + MAX(2.973 - 24, 0) + MAX(1.372 - 8, 0)] = 0$. Note that c_k (machine quantity × hours per shift) is found in Table 3.

The demand levels used during the experimentation were generated as follows. First, five sets containing 1, 2, 6, 10, and 15 make-items, respectively, were randomly generated. The demand of each item in these sets was taken from $U[1,100]$. This demand level represented almost a 10-fold increase of the demand levels considered in Hill and Raturi (1992). Then, the five sets of make-items were *MRP-exploded*. That is, one set was exploded at a time. As a result of these five MRP explosions, five sets of child-items with 4, 12, 142, 166, and 270 items, respectively, were generated. That is, the explosion of the first set with the one make-item generated four child-items. Similarly, the explosion of the second set with the two make-items generated 12 child-items, and so on and so forth. Although these five MRP explosions created child-items with a unique demand d_i, other demand levels were used by scaling d_i as follows: $0.01 \times d_i$, $0.1 \times d_i$, $1 \times d_i$, $10 \times d_i$, and $100 \times d_i$. These demand levels were necessary in order to test the ADH at much higher demand levels than the ones used in previous tests (a similar strategy can be seen in Blackburn and Millen (1984)).

Finally, the experimentation consisted of $5 \times 5 \times 5 \times R$ (i.e. 5 lot-sizing techniques, 5 demand levels,

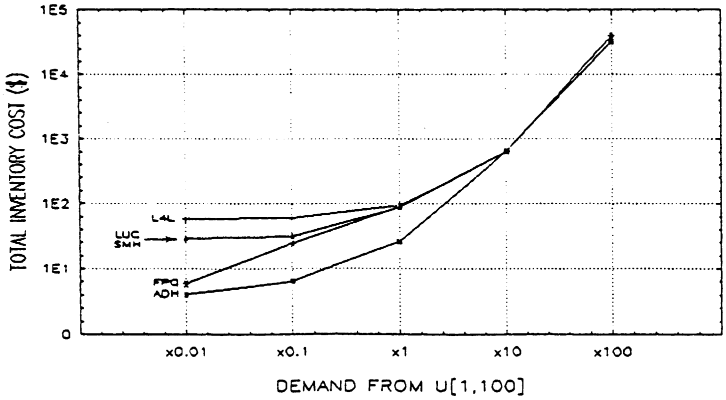

Fig. 3. Total inventory cost for four items.

5 sets of items, and R replications) runs. The value of R was 20 replications for 4, 12, and 142 items, but it was only five replications for 166, and 270 items because it was estimated that ADH would take 750 CPU hours (on a i486DX PC) to complete the 20 replications for 166 and 270 items. The computer program to conduct this experimentation was written in the PC-Fortran Ryan-McFarland programming language (Ryan-McFarland Corporation, 1984).

5.2. Computational results on TIC

Figs. 3–7 summarize the computational results on the TIC of the five lot-sizing techniques. These results indicate that for demand levels equal to $0.01 \times d_i$, $0.1 \times d_i$, and $1 \times d_i$, the TIC values of the ADH heuristic never exceeded the corresponding costs of the uncapacitated techniques. However, these results also show that for higher demand values (i.e. for values $10 \times d_i$ and $100 \times d_i$), the TIC values of capacitated and uncapacitated techniques were identical. That is, all five techniques performed similarly with the worst MRP lot-sizing performer (Haddock & Hubick, 1989): the L4L technique.

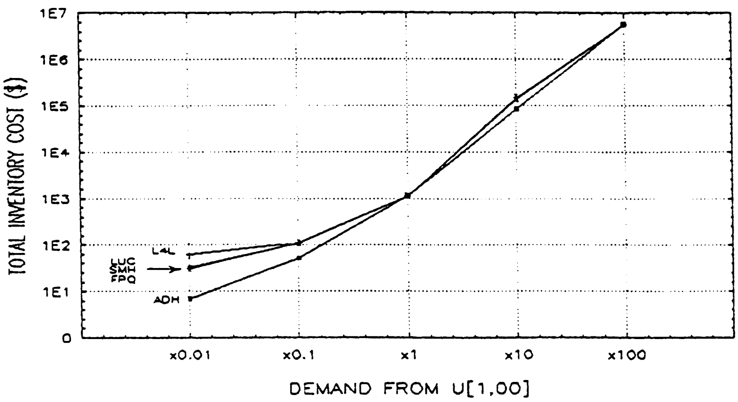

Fig. 4. Total inventory cost for 12 items.

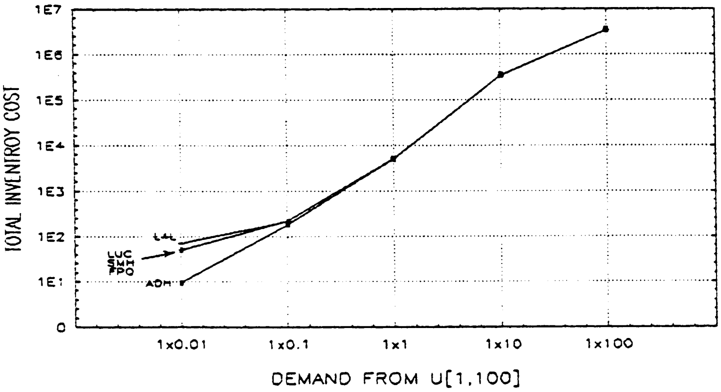

Fig. 5. Total inventory cost for 142 items.

Furthermore, The results in Figs. 5–7 show that when the number of items was increased to more than 12 items, then the TIC values of all five lot-sizing techniques converged more rapidly. (In the following figures, the notation $1En$ on the vertical axis stands for 1×10^n. For instance, $1E2 = 1 \times 10^2$.)

Relations (10) and (11), in Section 2.5, explain this rapid convergence of TIC values. First, relation (10) shows that a penalty cost P_w is incurred when the *infeasible condition* $L_w > C_w$ occurs (i.e. when the average load is greater than the available capacity). Furthermore, relation (11) defines L_w as a function of the number of items at machine w. Consequently, this infeasible condition $L_w > C_w$ is more likely to occur when the number of items is increased (assuming the same control conditions as in Hill and Raturi (1992) and Raturi and Hill (1988) of constant demands and costs). This is true because the second term of Eq. (11) is proportionally affected by the number of items. Thus, if $L_w > C_w$ occurs during the ADH's initial solution for item i (i.e. when $m_i = 0$ or equivalently when $n_i = 1$, which also corresponds to the L4L RI), then the TIC is penalized and thus is forced to accept that solution as its best solution for item i.

The results in this study show some limitations on the assumptions regarding the constant values of the holding costs, setup costs, and demands. In this study, however, these limitations opened the

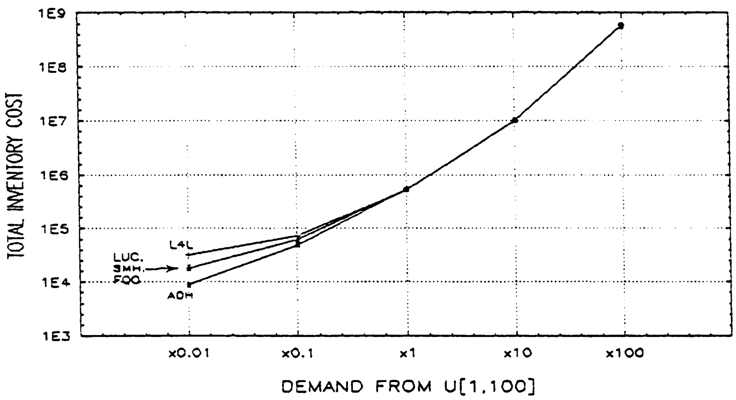

Fig. 6. Total inventory cost for 166 items.

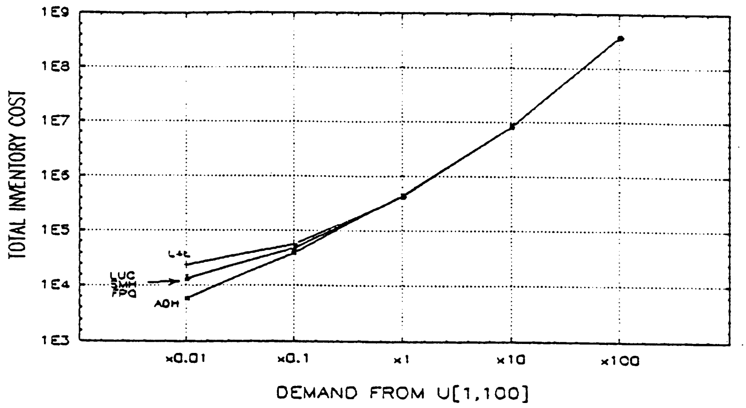

Fig. 7. Total inventory cost for 270 items.

possibility for comparing the near optimal (and also more complex and more time consuming) ADH lot-sizing technique with other more commonly used techniques. For example, a result of these comparisons suggests that RIs of fast techniques (such as the FPQ, LUC, and SMH) can be used as the initial solution of the ADH technique, which always starts with RI $n_i = 1$. This possibility becomes more attractive because of the very few capacity violations that were found when a single iteration of the ADH was applied on the RIs of uncapacitated techniques (see also Table 5). From Table 5 it can be seen that, on the average, the four uncapacitated techniques would deliver feasible initial solutions for the ADH technique 90% of the time.

6. Some analytical results on total inventory costs

This section presents some analytical derivations that determine when the previous four uncapacitated techniques may perform identically under certain conditions (e.g. different demand levels and/or increasing number of items). These derivations were motivated by the empirical results depicted in Figs. 3–7, which indicate that the five lot-sizing techniques delivered identical TICs. Furthermore, since the TIC values of these lot-sizing techniques may converge only to L4L, the worst performer, this technique was used as the method for the comparisons. On the other hand, analytical derivations for the ADH heuristic were not derived because its iterative nature made it impossible to find a closed form expression of the critical conditions under which the ADH could behave identically with the L4L.

Table 5
Frequency of capacity violations by uncapacitated techniques (note: 500 experiments were run for each technique)

	L4L	FPQ	LUC	SMH	Average
Number of capacity violations	52	45	47	47	47.75
Percentage of total	10.4	9.0	9.4	9.4	9.55

6.1. Conditions under which the SMH technique performs identically with the L4L

For the SMH (as defined in Section 2.2) to perform identically with the L4L, a reorder should occur at every period of the planning horizon. That is, every time window should be of size one period long. It can be easily verified that for the first window to be one period long, the following condition should be satisfied:

$$K_1 < \frac{\mathrm{K}_1 + H_1 d_2}{2}, \text{ or } H_1 d_2 > K_1. \tag{12}$$

where the term sub-indices indicate a period b ($1 \le b \le B$). That is, the average cost of the first period should be lower than the average cost of the first two periods. According to the SMH, if the previous condition is true, then the time window is comprised of only the first period.

The identification of the size of the second window is computed in a similar manner. Again, for the SMH to perform identically with the L4L, the second window should also be of size one period long (i.e. comprised by the second period only). The statement in relation (12) suggests that the necessary condition should be as follows:

$$K_2 < \frac{K_2 + H_2 d_3}{2}, \text{ or } H_2 d_3 > K_2. \tag{13}$$

That is, the average cost of the second period (which is also the first period for the second window) should be lower than the average cost of the second and third periods. Therefore, when the previous analysis is expanded for the third, fourth, etc., windows, then it is easy to realize that the SMH will perform identically with the L4L technique if the following condition is true:

$$H_b d_{b+1} > K_b \qquad \mathit{for}\ b = 1, 2, 3, \ldots B - 1. \tag{14}$$

6.2. Conditions under which the LUC technique performs identically with the L4L

The condition for LUC (defined in Section 2.3) to behave identically with the L4L technique means the LUC has to appear in every period. Like the SMH technique, the size of the window must be of one period long. It can be observed that if the first window is of size one, then the following condition must be satisfied:

$$\frac{K_1}{d_1} < \frac{K_1 + H_1 d_2}{d_1 + d_2}, \text{ or } H_1 > \frac{K_1}{d_1}. \tag{15}$$

That is, the average cost per unit in the first period should be lower than the average cost per unit of the first and second periods. The condition of the second window is computed similarly. That is, the second window must be one period long. Consequently, the following condition must also be satisfied:

$$\frac{K_2}{d_2} < \frac{K_2 + H_2 d_3}{d_2 + d_3}, \text{ or } H_2 > \frac{K_2}{d_2}. \tag{16}$$

That is, the holding cost for the second period (which is the first period for the second window) should be greater than the unit cost for the same period. Therefore, when the previous analysis is extended to all remaining periods in the planning horizon (i.e. for periods 3, 4, 5, …, B), it is easy to realize that the LUC will perform like the L4L technique if the following condition is satisfied:

$$H_p > \frac{K_p}{d_p}, \qquad \text{for } b = 1, 2, 3, \ldots, B. \tag{17}$$

6.3. Conditions under which the FPQ technique performs identically with the L4L

For the FPQ (defined in Section 2.4) to perform identically with the L4L technique, it must generate windows of size one period long. It can be observed that if the window size is one period, then Q^*/D should be in the range:

$$\max\left(1, \left\lfloor 0 < \frac{\bar{D}}{Q^*} < 2 \right\rfloor\right), \tag{18}$$

to guarantee a window size of one period long. That is, if the FPQ performs identically with the L4L, then Q^* must be within the range $0 < Q^* < 2D$. Thus, when Q^* is substituted with the EOQ formula, the condition for the FPQ to behave identically with the L4L is defined by:

$$\sqrt{\frac{2KD}{h}} < 2D, \quad \text{or} \quad K < 2hD. \tag{19}$$

It is important to mention here that the analytical derivations in Eqs. (14), (17) and (19) by no means replace the simulation experiments performed in this study. These derivations were needed to establish that the lot-sizing techniques may indeed perform identically under certain conditions, while the simulation experiments were needed to identify the existence of those conditions and to investigate how frequently these lot-sizing techniques may perform identically. Further, the simulation experiments were the only means to compare the ADH with the four uncapacitated techniques because the former is an iterative approach which made it impossible to derive closed form conditions.

7. Concluding remarks and summary

This study benchmarked the following five lot-sizing techniques: L4L, FPQ, LUC, SMH and ADH. Its goal was to investigate the behavior of the TIC of these techniques when this cost is a function of the RI. An experimental study was set up in order to investigate this cost behavior. The main conclusions of this investigation are as follows:

(1) For high demand levels, the uncapacitated lot-sizing techniques (i.e. L4L, FPQ, LUC, and SMH) delivered TICs which are comparable to the cost of the capacitated ADH technique.

(2) Moreover, as demand was increased, the TICs of all techniques converged into the TIC of the L4L technique. Thus, suggesting that for high demand levels, the L4L technique is a preferred technique mainly because of its mathematical simplicity.
(3) The ADH lot-sizing technique is recommended only for low demand levels and for small numbers of items in order to ensure acceptable CPU times.

The main contributions of this investigation are twofold. First, the results in this paper complement the previous research of the ADH lot-sizing technique (Hill & Raturi, 1992) by comparing its performance against the performance of four popular (Haddock & Hubick, 1989) lot-sizing techniques. Previous comparisons of this technique against the unpopular (Haddock & Hubick, 1989) branch-and-bound optimal method showed a deviation of only 0.32% (i.e. $<1\%$) from optimality for different problem sizes. However, these results are the first demonstration that for problems of large size the ADH (which is a near optimal) technique may behave identically with popular lot-sizing techniques. Furthermore, these results represent the antithesis of the assumptions in McKnew, Saydam and Coleman (1991), which state that simple lot-sizing techniques are poor performers with problems of reasonable size.

The second contribution corresponds to the analytical derivations of the conditions under which the uncapacitated techniques performed identically. These derivations complement the lower convergence point (LCP) in Karni (1986). The derivations in Karni (1986) also proved that some uncapacitated techniques (e.g. EOQ and UOQ) performed identically with the L4L for small RIs T, a parameter which is determined until the technique has been executed and the schedule is known. The analytical derivations in this study did not require the prior knowledge of such schedule, thus presenting an immediate advantage and a generalization over the derivations reported in Karni (1986).

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