Communication on the Paper "A Reference-Dependent Regret Model for Deterministic Tradeoff Studies"

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ABSTRACT

This communication focuses on a fundamental problem related to the recently introduced Reference-Dependent Regret Model (RDRM) [Kujawski, 2005] for deterministic multi-criteria decision making. In [Kujawski, 2005] it was asserted that the RDRM model satisfies three properties. The first of these properties, referred to as the "independence of dominated alternatives", seems to be an intuitive one. According to this property, the RDRM model preserves the ranking of two alternatives A_i and A_j with ranking $A_i \succ A_j$ when a new alternative dominated by A_i is introduced or an old alternative dominated by A_j is dropped. In this communication it is demonstrated algebraically and also by means of a numerical example that the RDRM model may fail to satisfy this property. The implication is that when the concepts of regret and/or rejoicing are considered and defined in terms of all the available alternatives in accordance with the RDRM, adding or dropping a dominated alternative can change the ranking of the alternatives and violate the independence of dominated alternatives property.

Key words: Regret theory, rank reversal, Pareto-optimal alternatives, utility theory, multi-criteria decision analysis.

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1. INTRODUCTION

Incorporating behavioral aspects, such as feelings of regret and rejoicing, into Multi-Criteria Decision Analysis (MCDA) has been an intriguing area. Among all the emotions, regret is the one that has received most of the attention. Savage [1951] first proposed the notion of regret more than 50 years ago. Later on, Loomes and Sugden [1982] and also Bell [1982] simultaneously proposed the regret theory for rational decision-making under uncertainty. In their regret theory (to be referred to as RT-B/LS), it is assumed that regret depends on the differences of the utilities of the chosen alternative and the forgone alternatives that were considered but were not chosen.

Recently, Kujawski [2005] argued that a person's level of regret often depends explicitly on the absolute values of the utilities of the chosen and forgone alternatives rather than simply the differences. He proposed the Reference-Dependent Regret Model (RDRM) to account for this behavior. More specifically, according to the RDRM the associated regret when choosing alternative A_i and forgoing alternative A_j under criterion C_k, denoted as $R(u_{ik}, u_{jk})$, is given by

$$R(u_{ik}, u_{jk}) = \begin{cases} G(1-u_{ik}) - G(1-u_{jk}), & u_{ik} < u_{jk} \\ 0, & \textit{otherwise.} \end{cases} \quad (1)$$

In Eq. (1), u_{ik} is the utility of alternative A_i for criterion C_k. The notation *G(.)* denotes the regret-building function. It measures the level of regret referenced to the maximum possible utility normalized to 1. The *G*-function used in the RDRM model is given as follows:

$$G(x) = \begin{cases} \dfrac{1}{1+(B/x)^{2*S*(B+x)}}, & x > 0, \\ 0, & \textit{otherwise.} \end{cases} \quad (2)$$

In Eq. (2), *B* and *S* are two parameters that depend on the regret attitude of the individual decision maker. The total level of regret for choosing A_i from a set *S* of *n* (where $n \geq 2$) alternatives with *m* criteria is given by

$$R_i^S = (\frac{1}{n-1})\sum_{k=1}^{m} w_k \sum_{j=1}^{n} R(u_{ik}, u_{jk}) \quad (3)$$

The final utility of alternative A_i given the set *S*, is calculated as follows:

$$U_i^s = \sum_{k=1}^{m} w_k u_{ik} - R_i^S = \sum_{k=1}^{m} w_k u_{ik} - \sum_{k=1}^{m} w_k (\frac{1}{n-1})\sum_{l=1}^{n} R(u_{ik}, u_{lk}) \quad (4)$$

In Eq. (4), the first term is the classical utility of alternative A_i and the second term is the anticipated regret for choosing alternative A_i. Finally, the alternatives are ranked by their overall utilities.

Furthermore, in [Kujawski, 2005], three properties were introduced which were argued to be necessary conditions that any effective MCDA model should satisfy. The first property is termed the "independence of dominated alternatives." According to this property, if a model ranks two alternatives A_i and A_j as $A_i \succ A_j$, then when adding a new alternative which is dominated by A_i or dropping an existing alternative which is dominated by A_j should preserve the initial ranking $A_i \succ A_j$. This property appears to be intuitive and it was thought that it had been proved to hold for the RDRM model [Kujawski, 2005]. However, as illustrated and proved in the next section, the RDRM model does not always satisfy this property.

2. RANK REVERSAL WITH THE RDRM MODEL

In this section, a randomly generated decision problem is used to illustrate how the RDRM model may fail to satisfy the property of independence of dominated alternatives stated in [Kujawski, 2005]. This is further investigated by analyzing the RDRM model algebraically.

2.1 An example of rank reversal with the RDRM model

This numerical example is defined for two alternatives with three decision criteria. The following matrix D represents the utilities of the two alternatives in terms of the three criteria while the row vector W represents the weights of the three criteria.

$$D = \begin{bmatrix} 0.6259 & 0.1098 & 0.7028 \\ 0.8793 & 0.0788 & 0.3143 \end{bmatrix}$$

$$W = [0.4149 \quad 0.4816 \quad 0.1035]$$

Furthermore, the two parameters used in the G-function are assumed to be $B = 0.60$ and $S = 4.00$.

When Eqs. (1), (2), and (3) are applied to compute the overall regret values associated for choosing each alternative and forgoing all the others the results are:

$$U_1 = P_1 - R_1 = 0.3853 - 0.0102 = 0.3802$$
$$U_2 = P_2 - R_2 = 0.4353 - 0.0836 = 0.3517.$$

Where P_i is the classical utility value of alternative A_i, R_i is the overall regret value for choosing alternative A_i and forgoing all the other alternatives, and U_i is the final utility value of alternative A_i after incorporating the effect of regret. Since $U_1 > U_2$, alternative A_1 is ranked higher than A_2; i.e., $A_1 \succ A_2$.

Next, a new alternative A_3 which is dominated by alternative A_1 is introduced. Please note that in this example it is a coincidence that A_2 also dominates A_3. The new decision matrix is given by

$$D = \begin{bmatrix} 0.6259 & 0.1098 & 0.7028 \\ 0.8793 & 0.0788 & 0.3143 \\ 0.2748 & 0.0458 & 0.1619 \end{bmatrix}.$$

The weight vector W remains the same as before. The same formulas are applied as before to rank the alternatives. The results are

$U_1 = P_1 - R_1 = 0.3853 - 0.0051 = 0.3802.$
$U_2 = P_2 - R_2 = 0.4353 - 0.0418 = 0.3935.$
$U_3 = P_3 - R_3 = 0.1528 - 0.4224 = -0.2696.$

Now $U_2 > U_1$, so $A_2 \succ A_1$. This result is different than the previous result of $A_1 \succ A_2$. A rank reversal has just occurred. Thus, the RDRM model failed the property of independence of dominated alternatives.

2.2 Mathematical Analysis

This section investigates mathematically why the RDRM model does not always follow the first property. Given a set S of n alternatives and m criteria, suppose that two alternatives, say alternatives A_i and A_j, are ranked as $A_i \succ A_j$. As described previously, the RDRM utility for alternative A_i and A_j are calculated as follows:

$$U_i^s = \sum_{k=1}^{m} w_k u_{ik} - R_i^S = \sum_{k=1}^{m} w_k u_{ik} - \sum_{k=1}^{m} w_k (\frac{1}{n-1}) \sum_{l=1}^{n} R(u_{ik}, u_{lk})$$

$$U_j^s = \sum_{k=1}^{m} w_k u_{jk} - R_j^S = \sum_{k=1}^{m} w_k u_{jk} - \sum_{k=1}^{m} w_k (\frac{1}{n-1}) \sum_{l=1}^{n} R(u_{jk}, u_{lk})$$

For convenience of discussion, let

$$R_i' = \sum_{k=1}^{m} w_k \sum_{l=1}^{n} R(u_{ik}, u_{lk}).$$

Then

$$U_i^s = \sum_{k=1}^{m} w_k u_{ik} - (\frac{1}{n-1}) R_i'.$$

Similarly, let

$$R_j' = \sum_{k=1}^{m} w_k \sum_{l=1}^{n} R(u_{jk}, u_{lk}).$$

Then

$$U_j^s = \sum_{k=1}^{m} w_k u_{jk} - (\frac{1}{n-1}) R_j' .$$

The difference between U_i^s and U_j^s is

$$U_i^s - U_j^s = (\sum_{k=1}^{m} w_k u_{ik} - \sum_{k=1}^{m} w_k u_{jk}) - (\frac{1}{n-1})(R_i' - R_j') . \quad (5)$$

Given that the two alternatives are ranked as $A_i \succ A_j$,

$$U_i^s - U_j^s > 0 \quad (6)$$

When introducing a new alternative A_k which is dominated by A_i, the value of R_i' remains unchanged while the value of R_j' may increase if A_k dominates A_j in terms of one or more criteria. Thus, in Eq. (5), the part $(R_i' - R_j')$ may become less than before. Meanwhile, the number of alternatives in the set S is increased by 1. Along with the above changes, if the original value of $(R_i' - R_j')$ is positive, the term $(\frac{1}{n-1})(R_i' - R_j')$ in Eq. (5) may become smaller than before. Then the inequality in Eq. (4) still holds. However, if the original value of $(R_i' - R_j')$ is negative, the term $(\frac{1}{n-1})(R_i' - R_j')$ may become larger than before. The inequality relation in Eq. (6) can be reversed and hence the ranking between A_i and A_j may be altered. This is how the RDRM model can fail to satisfy the property of independence of dominated alternatives.

3. ANOTHER WAY FOR MODELING REGRET

It may be questioned whether there is a way to avoid the above situation of rank reversal. Quiggin [1994] describes the problem where manipulation of a set of the alternatives may yield irrational choices as the ranking of the alternatives might be “money pumped”; i.e., the ranking of the alternatives is influenced by the introduction of dominated or non-Pareto optimal alternatives. In order to avoid being “money pumped”, Quiggin [1994] proposed that the measure of regret should satisfy a property called the Irrelevance of Statewise Dominated Alternatives (ISDA). This property is similar to the independence of dominated alternatives property proposed in [Kujawski, 2005]. In order to satisfy the ISDA property, Quiggin [1994] proved that regret must be determined solely by the best attainable outcome in each state of the world or equivalently the best performance value of each decision criterion in MCDA problems. This is in contrast with determining the regret associated with an alternative by considering the entire set of alternatives, like averaging the regret contributions that are formed when considering all available choice pairs. When this idea is applied to the study of how to model regret in MCDA problems, the regret associated with choosing one alternative and forgoing all the other alternatives is determined only by comparing the chosen criteria values with the best criteria values. Addition or deletion of dominated alternatives then cannot affect the regret levels of the other alternatives because the best criteria values will be the same as before under these changes.

In order to keep the ISDA property, it seems reasonable that regret should be measured only by the best possible value under each criterion. However, this may not make much sense as it is illustrated in the following hypothetical situation. Suppose that the following are the scores achieved by four students in some exam:

Students	Scores
A1	30
A2	32
A3	31
A4	100

In this example there is just one student who earned a high score (i.e., 100 points). Therefore, it is reasonable to expect that the student who earned only 30 points feels some but limited regret for not having achieved a higher score. This reaction may be rationalized because his/her performance is not as bad when it is compared to that of most of the other students. However, the same student may feel much stronger regret for scoring only 30 points if the scores of these students were as follows:

Students	Scores
A1	30
A2	98
A3	97
A4	100

This is understandable because in the new hypothetical example he/she is the only student who has achieved a very low score.

Therefore, intuitively in the previous example, it makes more sense to compute regret in terms of the entire set of alternatives. Thus, the concepts of regret and rejoicing may be more realistically expressed in terms of the criteria values of the entire set of alternatives, as given by Eq. (3), than in terms of only the best criteria values. However, on the other hand, if regret is computed by considering all the alternatives then adding or deleting a dominated alternative may alter the initial rankings and thus leaves the ranking of the alternatives open to manipulations like the "Money Pump".

4. CONCLUDING REMARKS

From the above discussion, one can see that if the concepts of regret and/or rejoicing are considered in the decision-making process and are defined in terms of all the available alternatives, the ranking of the alternatives when only the Pareto-optimal alternatives are used may be different than the ranking of the alternatives when some or all of the dominated alternatives are considered besides the Pareto-optimal ones. Adding or deleting a dominated alternative can change the ranking of any alternative because of the interdependence among them. Thus the RDRM does not satisfy the property of independence of dominated alternative that is proposed in [Kujawski, 2005]. The following question is still left answered and deserves further research: Is the property of independence of dominated alternatives a necessary condition that any effective MCDA model should satisfy or should its universality be considered suspect?

Problems with incorporating regret/rejoicing within the framework of classical utility are not unexpected because classical utility theory does not permit comparing differences in utility [Luce and Raiffa, 1957: 32]. The problem of formally modifying classical utility theory to incorporate rational psychological influences is challenging; but worth pursuing. An alternative approach proposed by Kujawski [2005] is to incorporate regret as an element of a cost-utility-regret analysis.

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