

Public Key Algorithms II

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These slides are available at:

<http://www.csc.lsu.edu/~durresi/csc4601-07/>



- Digital Signature Standard - DSS
- Zero-knowledge Proof Systems

Digital Signature Standard (DSS)

- By NIST, designed by NSA
 - Proposed in 1991, approved in 1994
 - DSA – algorithm
 - DSS – standard
- Related to El Gamal
- Speeded up for signer rather than verifier: smart cards
- Controversy RSA vs. DSS
 - When DSS was proposed a lot of investments were made in RSA
 - DSS a royalty free standard
 - DSS key size initially 512 later 1024
 - Designed by NSA

DSS Algorithm

- Generate public: p (512 bit prime) and q (160 bit prime): $p = k*q + 1$
 - This is a very expensive operation, but not often
- Generate public $g \neq 1$: $g^q = 1 \pmod p$
 - Take a random $h > 1$ get $g = h^{(p-1)/q}$
 - If $g = 1$, chose another h
- Choose long-term public/private key pair $\langle T, S \rangle$, with random S
 - $T = g^S \pmod p$ for $S < q$

DSS Algorithm (Cont'd)

- Choose a per message public/private key pair $\langle T_m, S_m \rangle$ with random S_m for message m
 - $T_m = (g^{S_m} \pmod p) \pmod q$
 - Calculate $S_m^{-1} \pmod q$, so it won't need to be done in real time when signing a message
- Calculate a digest of the message $d_m = \text{SHS}(m)$
- SHS – a hashing function recommended by NIST for use with DSS. SHS hashes to 160 bits

DSS Algorithm (Cont'd)

- Signature $X = S_m^{-1}(d_m + S T_m) \pmod q$
- Transmit m, T_m, X
- Public key information T, p, q , and g are known beforehand
- Verify:
 - Compute $X^{-1} \pmod q$
 - Compute d_m
 - $x = d_m X^{-1} \pmod q$
 - $y = T_m X^{-1} \pmod q$
 - $z = (g^x T^y \pmod p) \pmod q$
 - if $z = T_m$, the signature is verified

Why Is DSS Secure

- ❑ No revealing of private key S
- ❑ Can't forge a signature without S
- ❑ No duplicate messages with matched signature
- ❑ Can't be tempered
- ❑ Need a per-message secret number (S_m)!
 - If S_m known, S can be computed
 - Two messages sharing the same S_m can reveal S_m , and thus S

Per message Secret Number

- ❑ How is the private key exposed if the secret number per message is known?
- ❑ If S_m is known, one can compute:
 - $(X_m S_m - d_m) T_m^{-1} \bmod q = S \bmod q$
- ❑ How is the private key exposed when two messages share the same secret number?
 - If m and m' are signed using the same S_m
 - One can compute:
 $(X_m - X_{m'})^{-1} (d_m - d_{m'}) \bmod q = S_m \bmod q$
- ❑ How to generate random numbers?

How Secure are RSA and Diffie-Hellman ?

- ❑ The security of RSA is based on the difficulty of factoring
- ❑ The security of Diffie-Hellman is based on the difficulty of resolving discrete log problems
- ❑ It has been proved that they are equivalently difficult
- ❑ The best known algorithms to resolve them are subexponential but superpolynomial
- ❑ That's why a larger key size is required (1024bits) compared to secret key (80 bits)

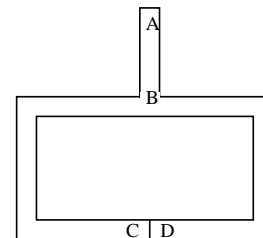
Elliptic Curve Cryptography ECC

- ❑ No known subexponential algorithms to break ECC
- ❑ So it is secure with much smaller key – improve performance
- ❑ An elliptic curve is a set of points satisfying an equation of the form:
 $y^2 + ax + by = x^3 + dx + e$
- ❑ Mathematical operation on two points in the set will always produce a point in the set

Zero-Knowledge Proofs

- ❑ Alice: "I know the ingredients in McDonald's secret sauce"
- ❑ Bob: "No, you don't"
- ❑ Alice: "Yes, I do"
- ❑ Bob: "Prove it"
- ❑ Alice: "All right. I'll tell you." She whispers in Bob's ear"
- ❑ Bob: "Now I know it too. I will post in my web page"
- ❑ Alice: "Oops"

The Zero-knowledge Cave



The Zero-knowledge Cave

- ❑ 1. Bob stands at point A
- ❑ 2. Alice walks all the way into the cave, either to point C or D
- ❑ 3. Bob walks to point B
- ❑ 4. Bob asks Alice to
 - Come out of the left passage
 - Come out of the right passage
- ❑ 5. Alice complies by using the magic word to open the door C or D
- ❑ 6. Bob and Alice repeat steps 1-5 n times.

Zero-knowledge Proof Systems

- ❑ Prove knowledge without revealing it
- ❑ RSA signatures
- ❑ Graph isomorphism: rename vertices
 - Alice: graph A and $B \sim A$
 - Public key: graphs A, B
 - Private key: mapping between vertices
 - Alice: create G_i and sends to Bob
 - Bob \rightarrow Alice: how did A or $B \rightarrow G_i$?
 - Zero-knowledge: Bob knows nothing about the mapping between A and B
 - Fred can generate G_i from either A or B , but not both

Zero-knowledge Proofs: Fiat-Shamir

- ❑ Alice: public key $\langle n, v \rangle$, $n=pq$, private key s
 - Alice selects random number s , and computes $v=s^2 \pmod n$
- ❑ Alice chooses k random numbers r_1, \dots, r_k
- ❑ Alice sends $r_i^2 \pmod n$
- ❑ Bob chooses a random subset 1 of r_i^2 , the rest is subset 2
 - For subset 1, Alice sends $sr_i \pmod n$, and Bob verifies $(sr_i)^2 \pmod n = vr_i^2 \pmod n$
 - For subset 2, Alice simply sends $r_i \pmod n$
- ❑ Finding square root mod n is hard

Fiat-Shamir (cont.)

- ❑ Suppose Fred wants to impersonate Alice
- ❑ Fred can compute *squares mod n* but cannot take *square roots mod n*
- ❑ Fred can send random r and compute r^2 : so he can give correct answers for subset 2, but not for subset 1
- ❑ So what the purpose of subset 2? Why isn't the protocol simply that Alice sends pairs (r_i^2, sr_i) ?

Fiat-Shamir (cont.)

- ❑ If only (r_i^2, sr_i) was used
 - If Fred has overheard Alice providing (r_i^2, sr_i) , he can impersonate Alice.
- ❑ But because both subsets are needed
 - Even if he knows subset 1 + answer and subset 2 + answer, the probability of having the new subset 1' equal to subset 1 is very small
 - For each I the probability is 50%, for the whole subset (30) no chance to impersonate, on in ten billion

Summary

- ❑ Digital Signature Standard - DSS
- ❑ Zero-knowledge Proof Systems

