1. Consider a standard programming language (say, like C or Pascal). Suppose we have defined the context sensitive syntax of this language using attribute grammars in the same manner as what we discussed in class. Suppose we now wish to add the requirement that all variables must be initialized or assigned a value before they are used (without requiring syntactically that every declaration contains an initializer). E.g., the Java specification has such a requirement. Is it possible to check this condition in the context-sensitive syntax of the language (using attributes such as `symtab`, `tbl`, etc.)? If yes, explain briefly how you would modify the grammar to check this condition. If no, explain how a code-generation attribute grammar would need to be modified such that this condition is checked at runtime. Such a check could be performed independently of the scope rules of the language, so you can restrict yourself to the simple languages discussed in class.

2. For each of the following assertions, state whether it is operationally valid or not. Justify your answers briefly. If for either of these assertions you cannot decide whether it is operationally valid or not, explain why you are not able to decide.

   (a) \{x = 0\} while x < 10 do x := x - 1; end \{x = x + 1\}
   (b) \{x = 0\} while x < 10 do x := x + 1; end \{x = x + 1\}
   (c) [x = 0] while x < 10 do x := x - 1; end [x = x + 1]
   (d) [x = 0] while x < 10 do x := x + 1; end [x = x + 1]

3. Consider the following statement:

   \[
   \langle \text{letstmt} \rangle ::= \text{let int } \langle \text{id} \rangle := \langle \text{intexp} \rangle \text{ in } \langle \text{stmtseq} \rangle \text{ end}
   \]

   When executing this statement, we first evaluate the integer expression \langle \text{intexp} \rangle. We then bind the variable \langle \text{id} \rangle to the value of this expression, and then evaluate the statement sequence \langle \text{stmtseq} \rangle.

   Semantic constraints: The binding in the let-statement shadows any outer declaration of the variable \langle \text{id} \rangle. I.e., any occurrence of \langle \text{id} \rangle in \langle \text{stmtseq} \rangle refers to the variable from the let-statement. The scope of this variable is only \langle \text{stmtseq} \rangle. I.e., any occurrence of \langle \text{id} \rangle in \langle \text{intexp} \rangle, in a post-condition of the let-statement, or in a statement following the let-statement refers to some outer declaration of \langle \text{id} \rangle. There are no restrictions on the use of variable \langle \text{id} \rangle in \langle \text{stmtseq} \rangle. In particular, it is possible to assign a new value to \langle \text{id} \rangle. The let-statement is evaluated strictly. I.e., if \langle \text{intexp} \rangle does not terminate (assuming that there can be function calls in \langle \text{intexp} \rangle), then the let-statement does not terminate.

   Propose an axiom or a proof rule for the let-statement. Justify your proposal (informally). If you cannot formally define the semantics according to all the semantic constraints above, define it as good as you can and explain in English what’s missing.
4. Consider Dijkstra’s guarded loop:

\[
\langle c \rangle ::= \text{do } \langle be \rangle_1 \rightarrow \langle c \rangle_1 \cap \langle be \rangle_2 \rightarrow \langle c \rangle_2 \cap \langle be \rangle_3 \rightarrow \langle c \rangle_3 \text{ od}
\]

where \text{do} and \text{od} are keywords, \langle be \rangle_i are boolean expressions and \langle c \rangle_i are commands. (Dijkstra’s guarded loop has arbitrarily many clauses; we’ll restrict ourselves to exactly three for simplicity.)

The guarded loop is a generalization of the while loop with multiple conditions (\langle be \rangle_1 \ldots \langle be \rangle_3) and multiple loop bodies (\langle c \rangle_1 \ldots \langle c \rangle_3). The boolean expressions are guards for the commands that follow them. A command \langle c \rangle_i can only be executed if its guard \langle be \rangle_i is true.

Informally, each loop iteration of the guarded loop is executed as follows. All the boolean expressions (guards) are evaluated. If none of the guards \langle be \rangle_i are true, none of the commands \langle c \rangle_i are executed and the loop terminates. If one guard is true, the corresponding command is executed. If more than one guard is true, one of the commands whose guard is true is chosen nondeterministically (at random) to be executed. After execution of the selected command, execution proceeds to the next loop iteration.

The special case in which two of the guards are always false is equivalent to the while loop.

Propose an axiomatic proof rule corresponding to this command. Do not assume a certain deterministic evaluation order. Explain your proof rules briefly.

5. The following program takes a positive integer \(n\) as input, computes the factorial function \(n!\), and returns the result as the value of \(f\). Recall that \(0! = 1\) and \(n! = 1 \cdot \ldots \cdot n\) for \(n > 0\).

Find the loop invariant and a termination function and prove the total correctness of this program.

\[
\begin{align*}
\{ n \geq 0 \} \\
& \quad f := 1; \\
& \quad i := n; \\
& \text{while } i > 1 \text{ do} \\
& \quad \quad f := f \times i; \\
& \quad \quad i := i - 1 \\
& \text{end} \\
& \{ f = n! \}
\end{align*}
\]

Clearly separate the proof into the three parts before, in, and after the while loop, and indicate the verification conditions resulting from the rule of consequence.