# CSC 4356
# Interactive Computer Graphics
## Lecture 14: Illumination (Part 1)

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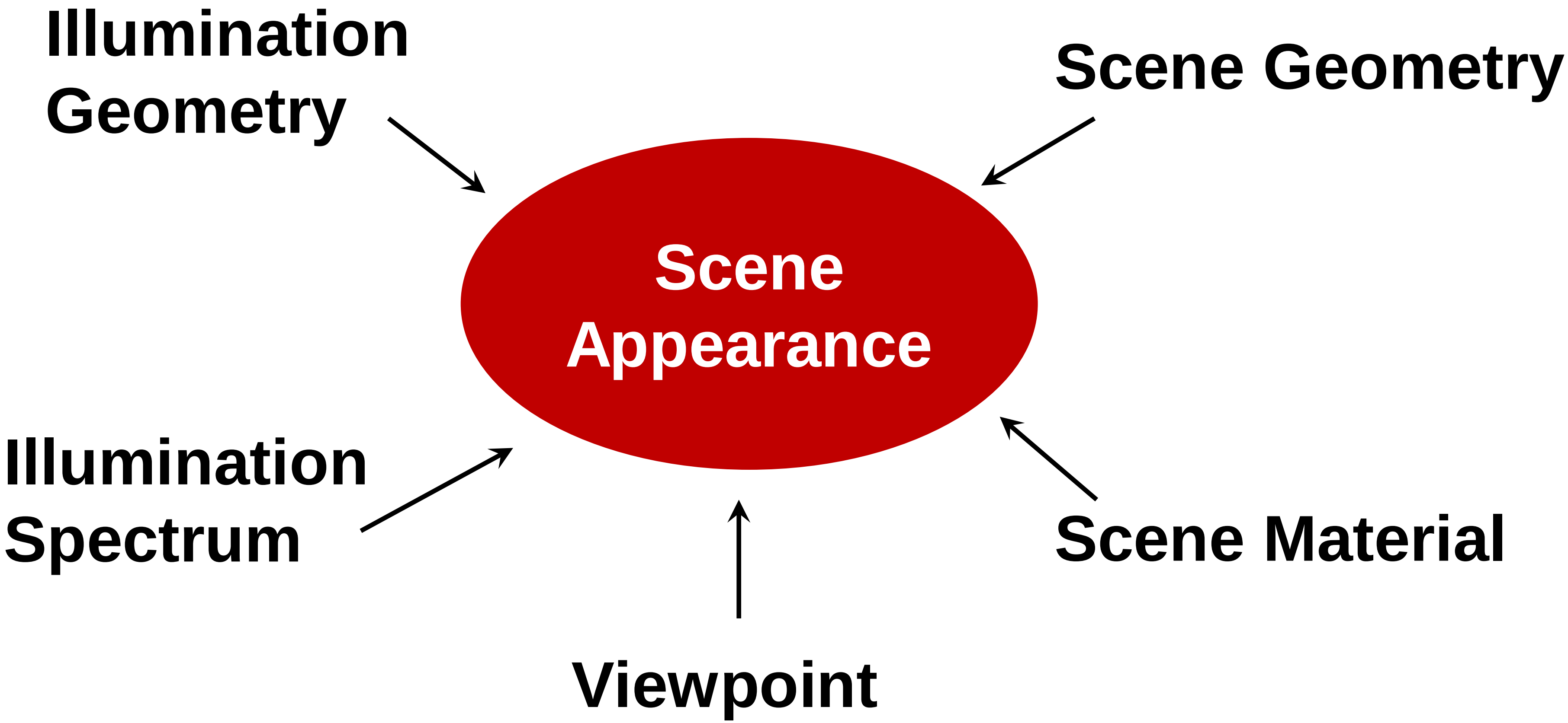
Tue & Thu: 10:30 - 11:50am
218 Tureaud Hall

# Lecture 14 Illumination

- So far, we have talked a lot about geometry
- This time, let’s shed some light …
  - Reading: Chap 17

# Appearance of An Scene



# Example: Time-lapse Photo

- Daytime

*Image from rumble.com*

# Example: Time-lapse Photo

- Sunset

*Image from rumble.com*

# Example: Time-lapse Photo

- After Sunset

*Image from rumble.com*

# The Dress

*Image from Twitter*

# Two Interpretations

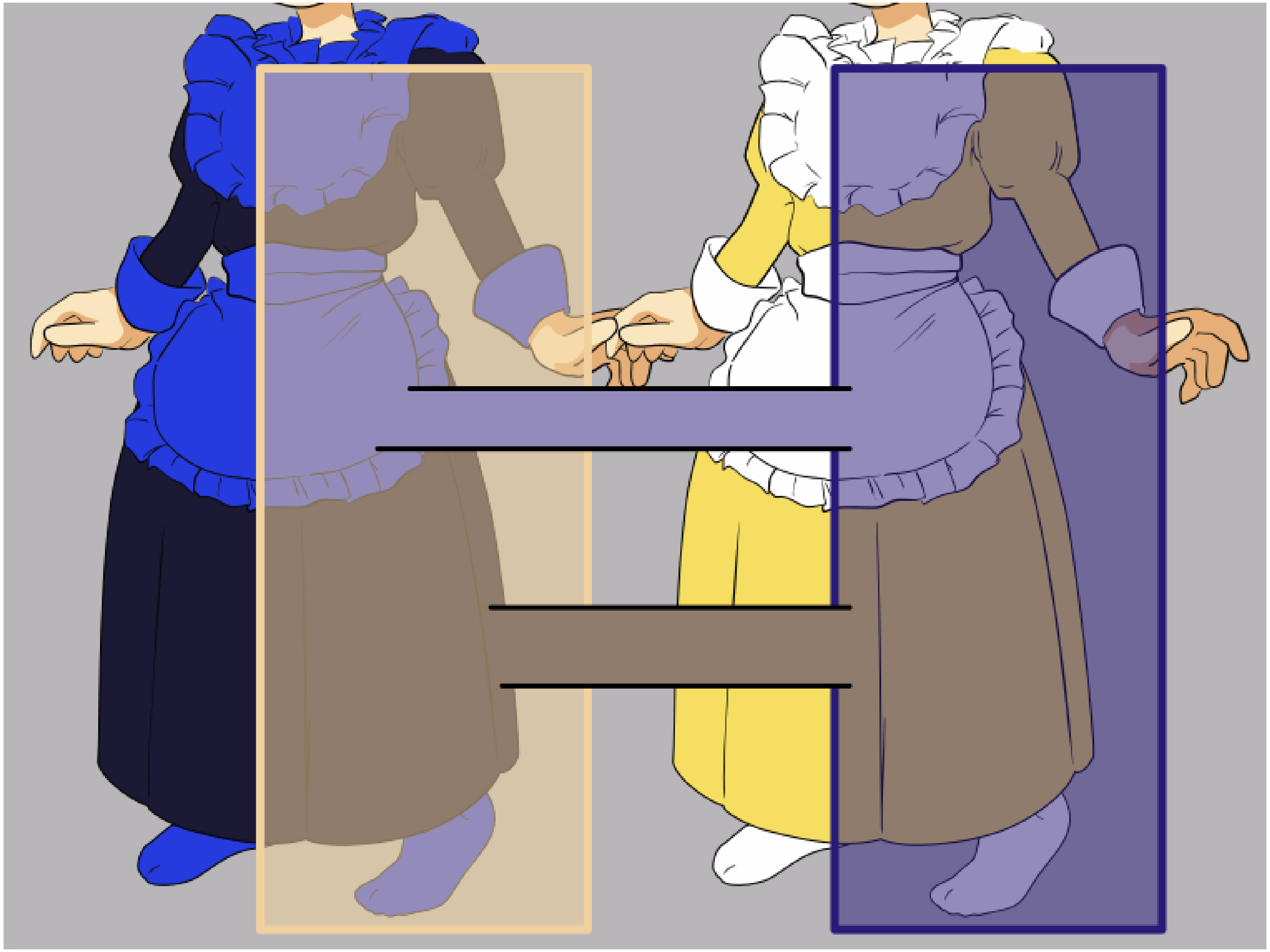


*Image from Wikipedia*

# What is Light?

- Light is electromagnetic radiation that is visible to eyes
  - 3D position
  - Propagation direction
  - Wavelength
  - …

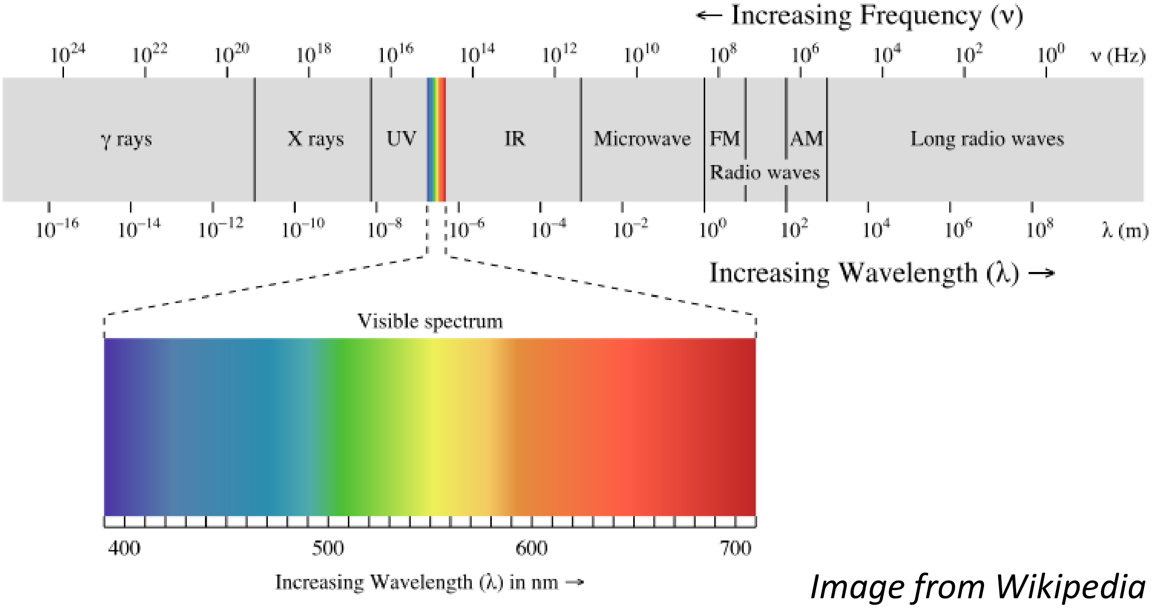
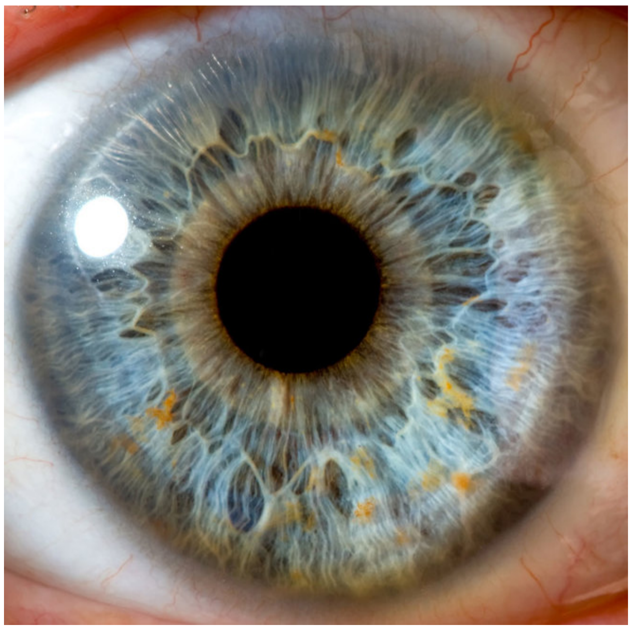


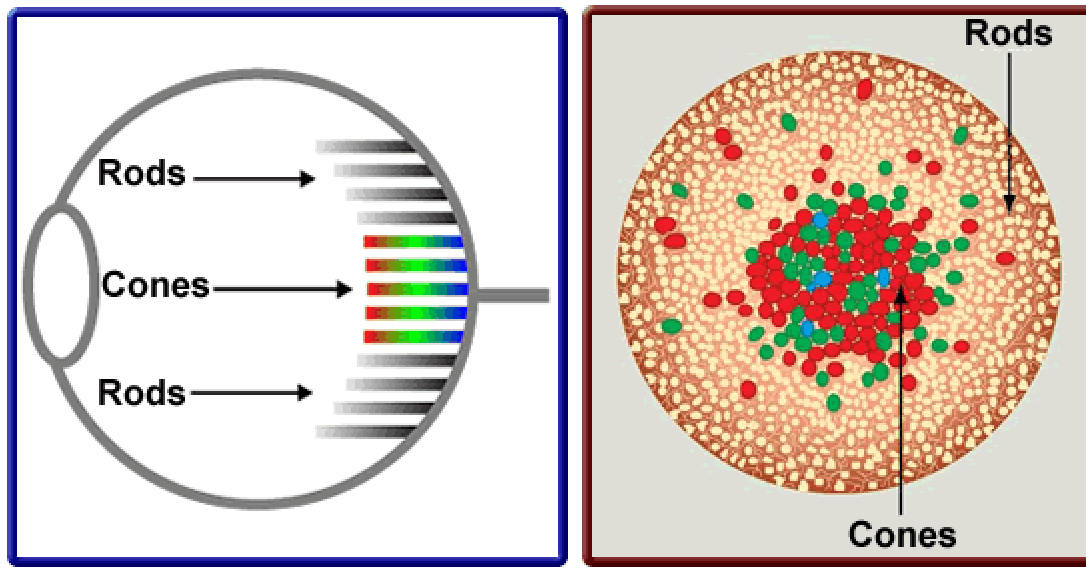

*Image from Wikipedia*

- We assume objects are large…
  - Much larger than the visible light wavelength
  - We don't consider interference and diffraction

# How to Measure Light?

- Human eyes

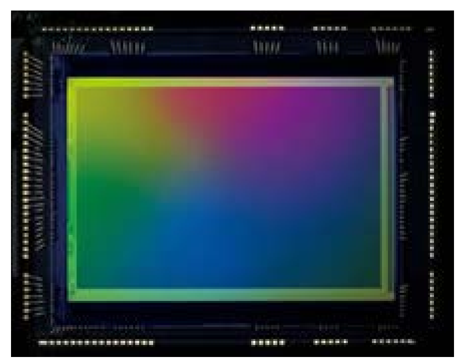




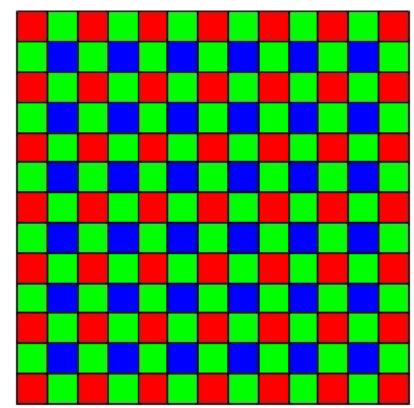

**Retina**

# How to Measure Light?

- Camera

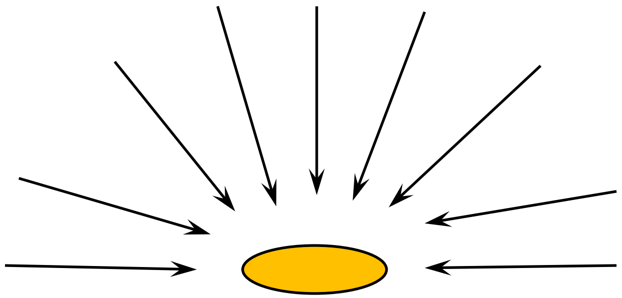
Sensor



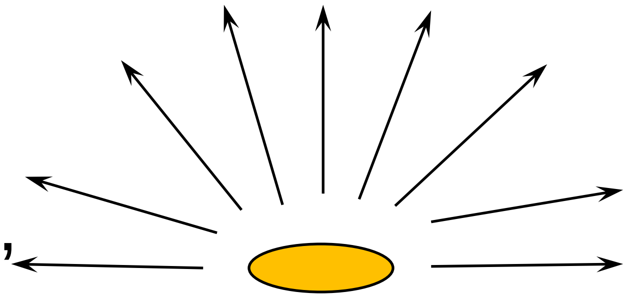
Bayer Filter

# Measure Light: Mathematical Models

- Irradiance E
  – Light power per area that ***arrives*** at surface from all visible direction



- Radiosity B
  – Light power per area that ***leaves*** a surface in all directions
  – Also called “Radiant Exitance”



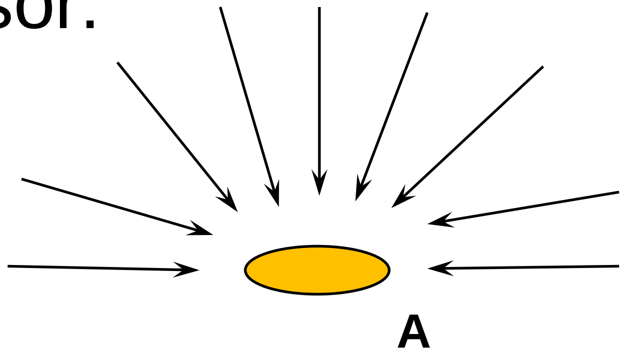
# Irradiance

- Power of light is measured by flux in radiometry
  - Unit: Lumen (lm) or Watts (W)
- Given a sensor with area A, we can consider the average flux over the entire sensor area:

$$\frac{\Phi}{A}$$

- Irradiance (E) is given by taking the limit of area at a point on the sensor:

$$E(\mathrm{p}) = \lim_{\Delta \to 0} \frac{\Delta\Phi(\mathrm{p})}{\Delta A} = \frac{\mathrm{d}\Phi(\mathrm{p})}{\mathrm{d}A}$$



# Beam Power in Irradiance

- Let’s consider a beam of light with flux ϕ incident on surface with area A

$$E = \frac{\Phi}{A}$$

$$\Phi = EA$$

# Tilted Surface

- Let's consider a beam of light with flux ϕ incident on an tilted surface with area A'


$$A = A' \cos\theta$$

**Here A is the projected area of surface relative to the direction of beam**

# Foreshortening

- Irradiance is proportional to cosine of the angle between light direction and surface normal


$$A = A' \cos\theta$$

$$E = \frac{\Phi}{A'} = \frac{\Phi \cos\theta}{A}$$

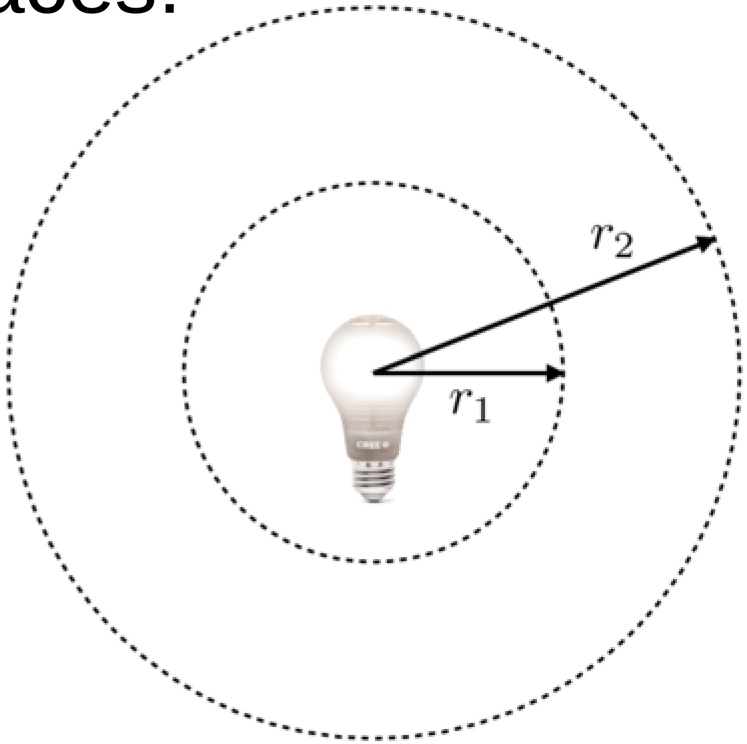
# Distance Attenuation

- Assume a light source emitting flux in a uniform angular distribution
- Consider two sphere surfaces:

$$E_1 = \frac{\Phi}{4\pi r_1^2} \rightarrow \Phi = 4\pi r_1^2 E_1$$

$$E_2 = \frac{\Phi}{4\pi r_2^2} \rightarrow \Phi = 4\pi r_2^2 E_2$$

$$\frac{E_2}{E_1} = \frac{r_1^2}{r_2^2}$$



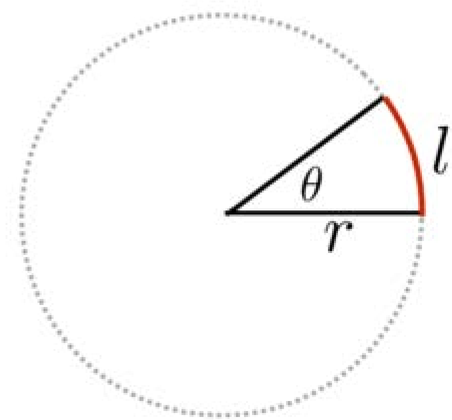
# Radiance

- Irradiance doesn’t tell us where the light comes from
  - Integrate light over all directions
- Radiance L
  - Power of light per area in a given direction
  - Since light is a cone, direction is defined in solid angle
  - Can be both incoming and outgoing

# Solid Angle

- Angle: ratio of subtended arc length on a circle to radius

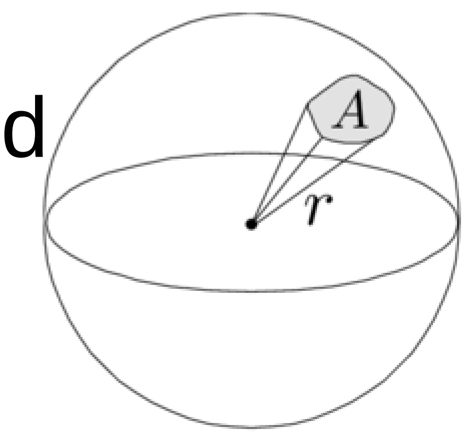
$$\theta = \frac{l}{r}$$




Circle has 2π radians

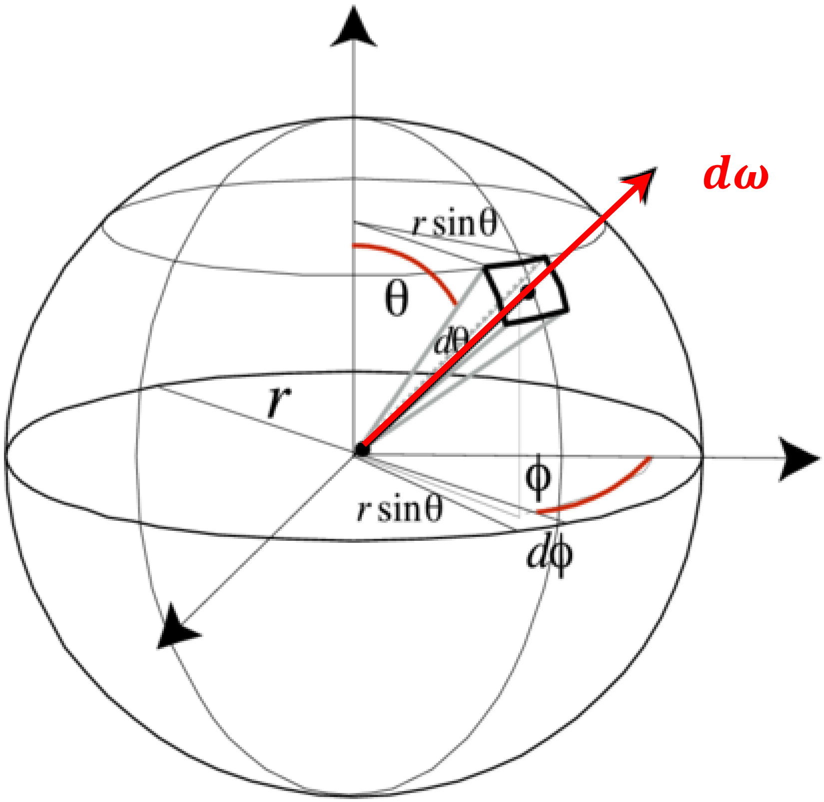
- Solid angle: ratio of subtended area on sphere to radius squared

$$\Omega = \frac{A}{r^2}$$




Sphere has 4π steradians

# Differential Solid Angle




$$\mathrm{d}A = (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi)$$
$$= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

**Entire Sphere:**

$$\Omega = \int_{S^2}\mathrm{d}\omega$$
$$= \int_0^{2\pi}\int_0^{\pi}\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$
$$= 4\pi$$

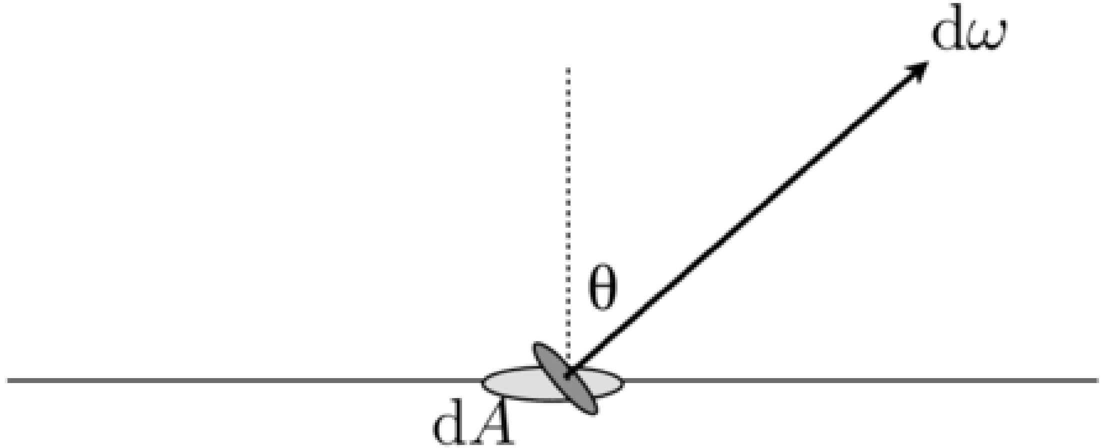
# Surface Radiance

- Surface Radiance (L) is light power per area per solid angle leaving the surface
  - Irradiance per solid angle
  - Unit: $W/(m^2 \cdot sr)$


$$L(p,\omega) = \frac{dE(p)}{d\omega} = \frac{d^2\Phi(p)}{dA\,d\omega}$$

# Surface Radiance

- If the surface is not oriented in the ray direction:

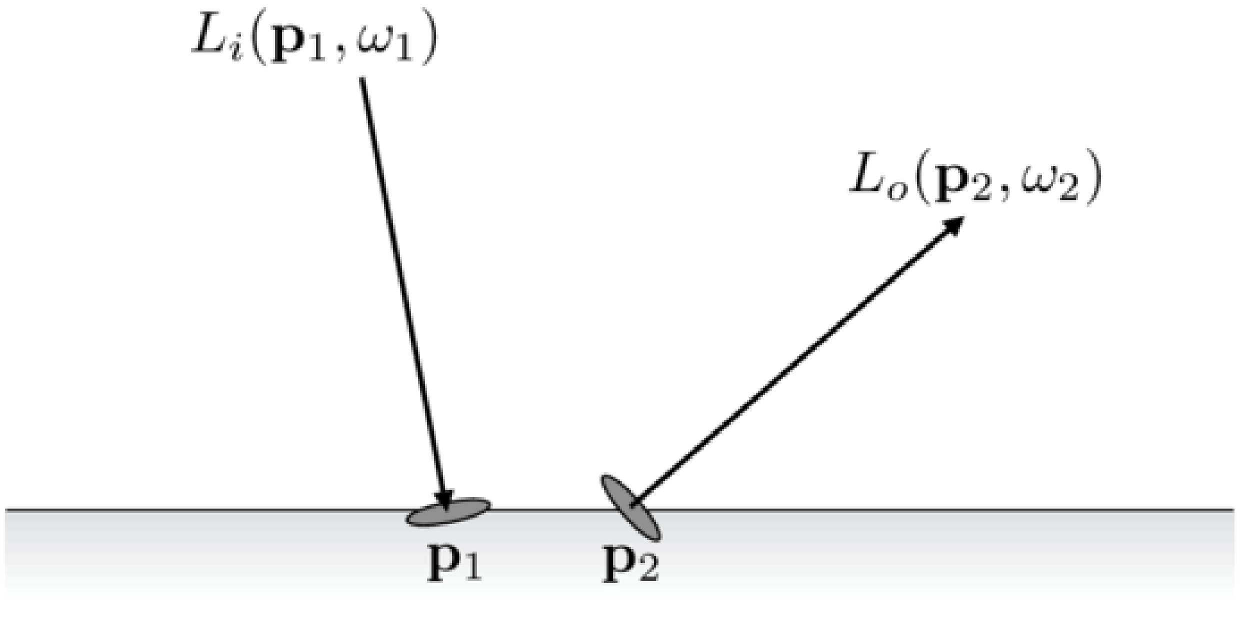



$$L(p,\omega) = \frac{d^2\Phi(p)}{dA\,d\omega\cos\theta}$$

# Radiance: Incoming vs. Outgoing

- Radiance could be in both directions
- General on the same surface point p:

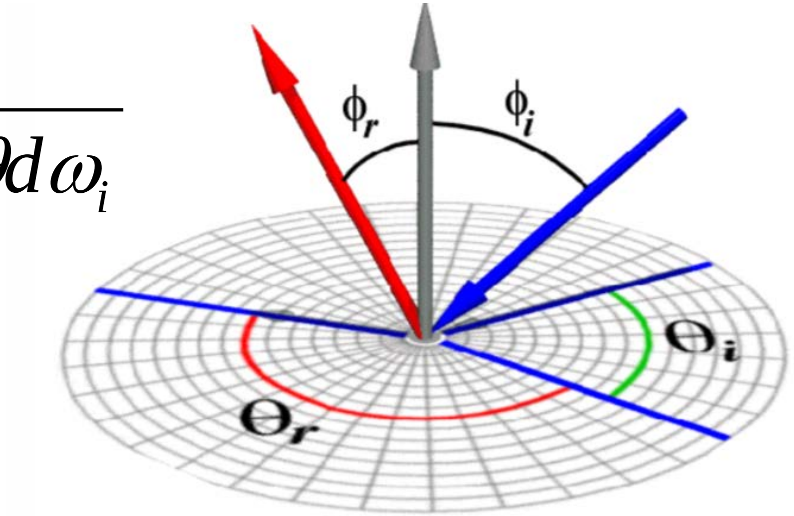
$$L_i(\mathbf{p}, \omega) \neq L_o(\mathbf{p}, \omega)$$



# BRDF

- **B**i-directional **R**eflectance **D**istribution **F**unction describes the transport of incoming light to outgoing light (surface radiance)
  - Ratio between outgoing surface radiance and incoming irradiance
  - Characterize surface reflectance/material

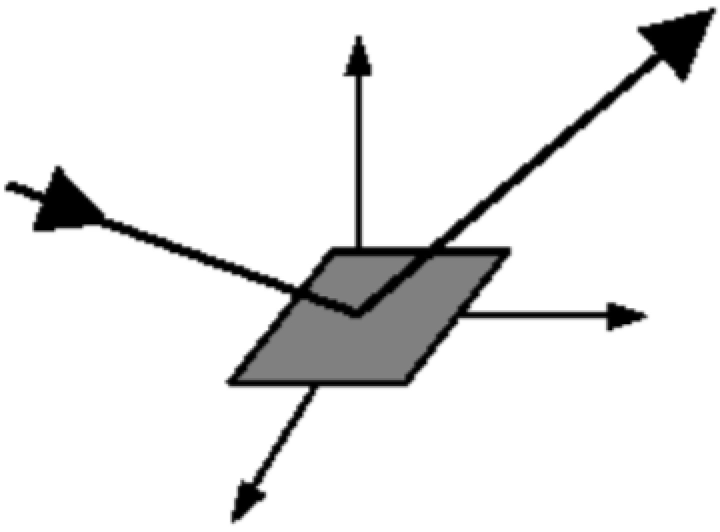
$$\rho(\theta_r, \phi_r, \theta_i, \phi_i) = \frac{L(\theta_r, \phi_r)}{L(\theta_i, \phi_i)\cos\theta d\omega_i}$$



# BRDF Property

- Reciprocity principle

$$\rho(\theta_r, \phi_r, \theta_i, \phi_i) = \rho(\theta_i, \phi_i, \theta_r, \phi_r)$$



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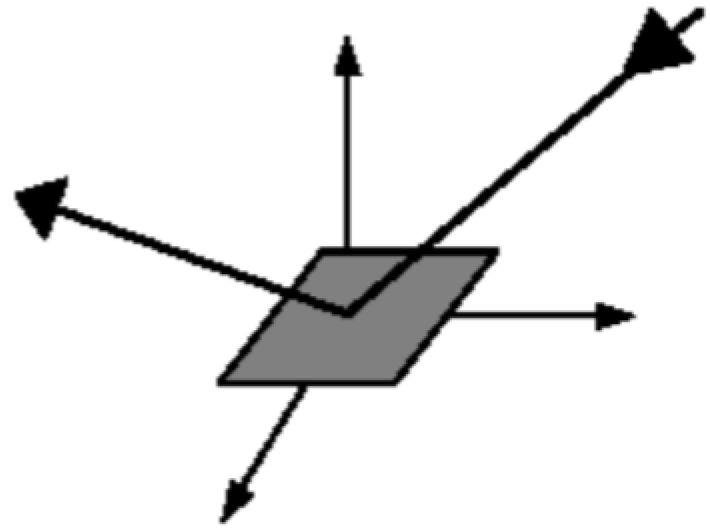
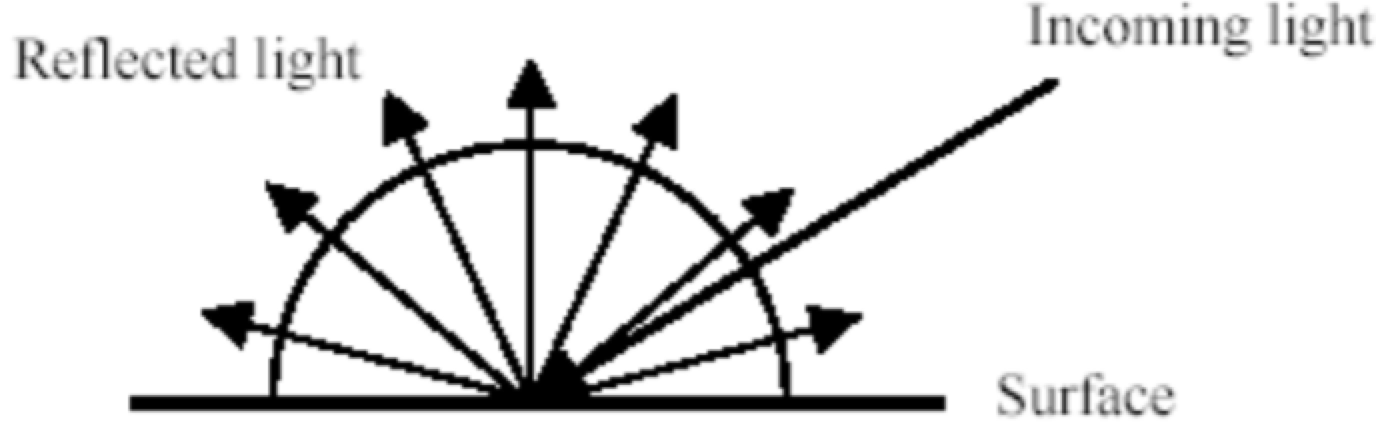


# BRDF Property

- Energy conservation

$$\int_{H^2} \rho(\theta_r, \phi_r, \theta_i, \phi_i) \leq 1$$

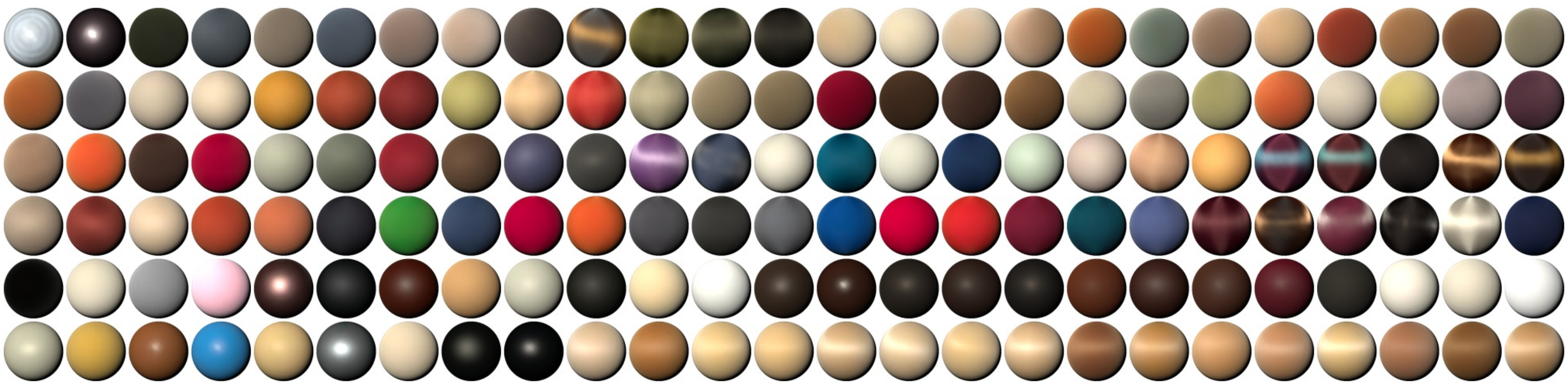
where $H^2$ is a hemisphere



# Measuring BRDF

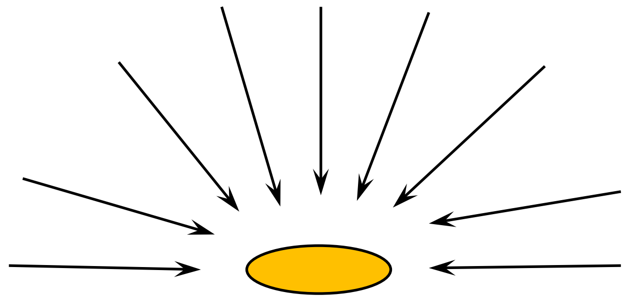
- Goniophotometer
  - 4 degree-of-freedom gantry
  - Measure one incoming/ outgoing light pair at time
  - Slow
  - Accurate

# BRDF Examples

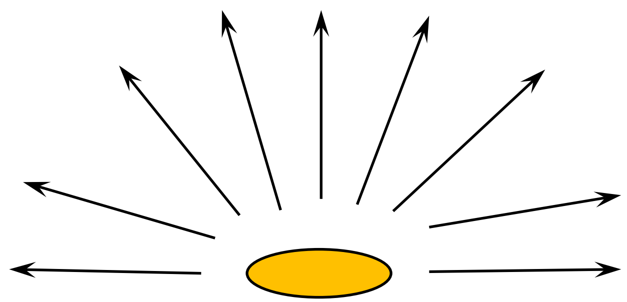


# What are we measuring?

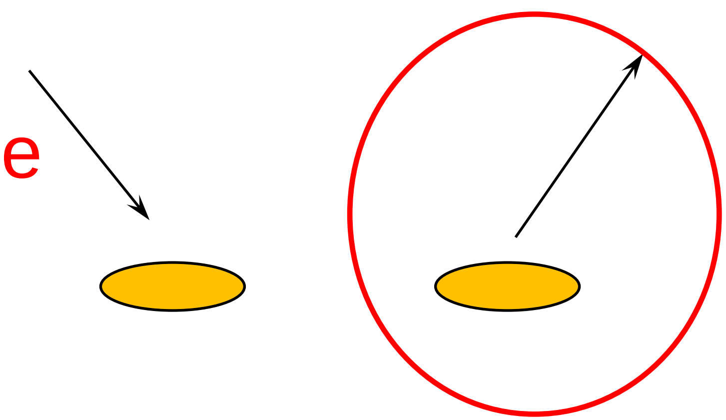
- Irradiance



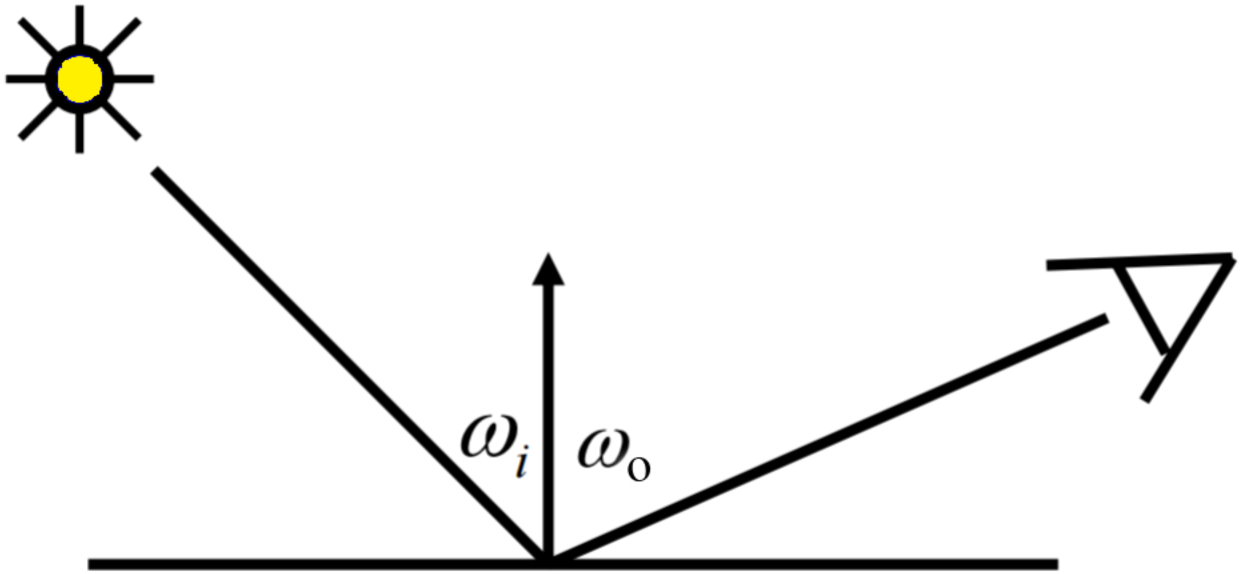
- Radiosity



- Radiance
  - Reflected radiance



# Reflectance Equation




$$L_r(\omega_r) = L_i(\omega_i)\, f(\omega_i, \omega_r)(\omega_i \cdot n)$$

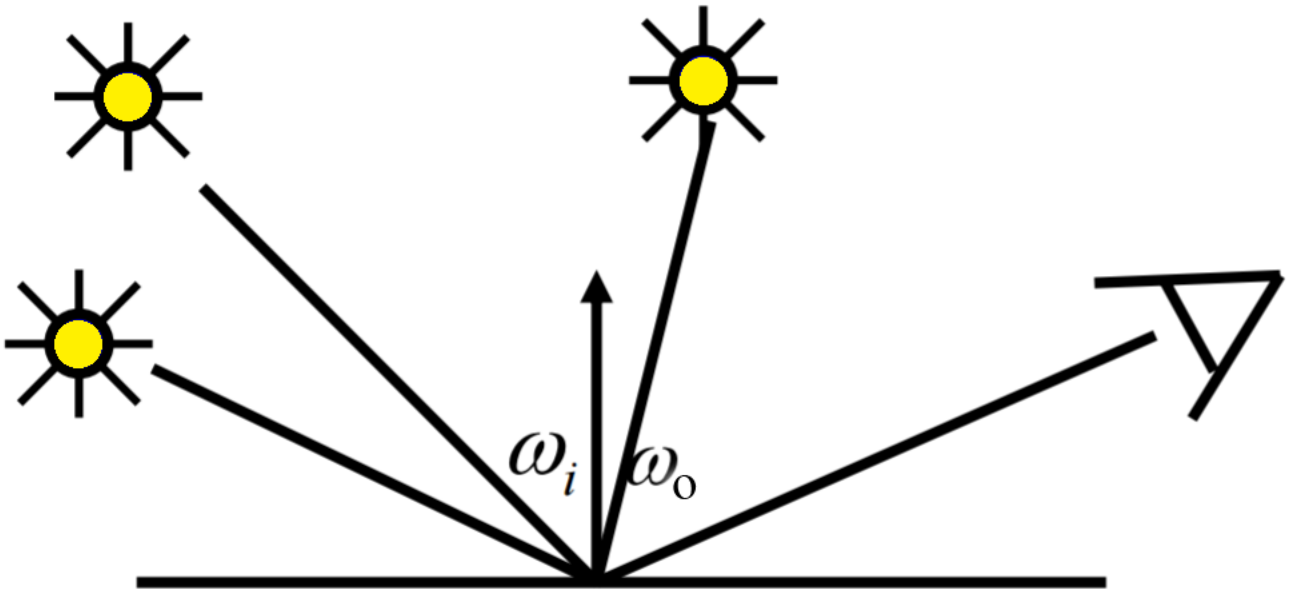
**Reflected Radiance (Output Image)**

**Incident Radiance (from light soure)**

**BRDF**

**Cosine of Incident Angle**

# Reflectance Equation




Sum over all light sources

$$L_r(\omega_r) = \sum_i L_i(\omega_i) f(\omega_i, \omega_r)(\omega_i \cdot n)$$

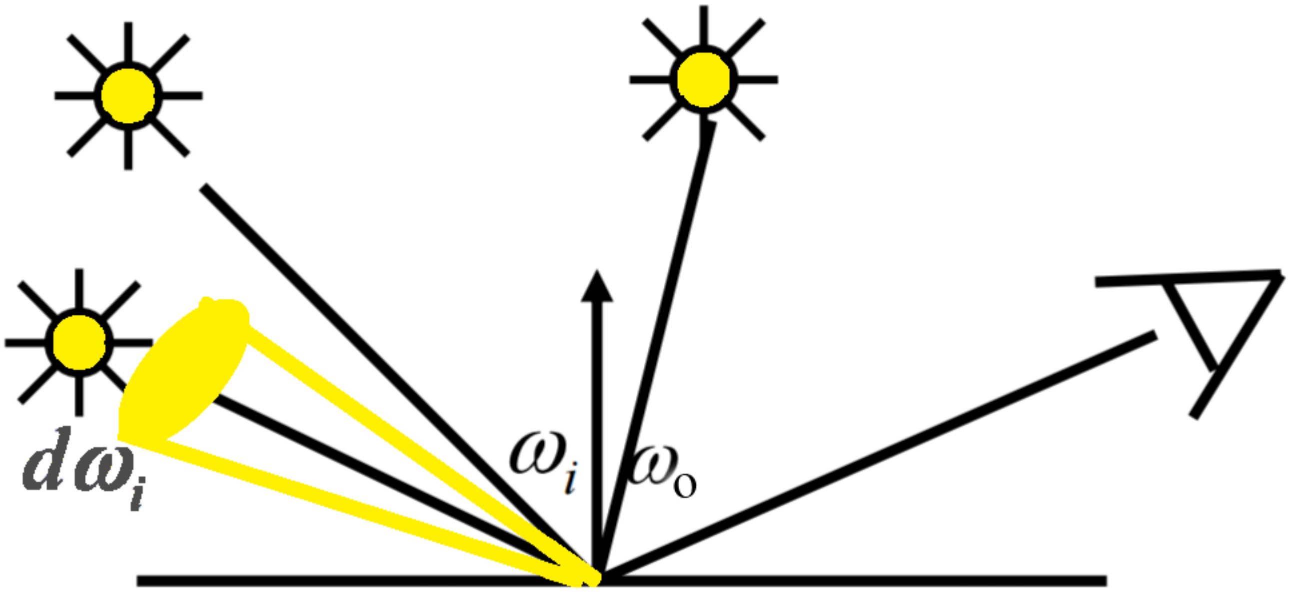
**Reflected Radiance (Output Image)**

**Incident Radiance (from light soure)**

**BRDF**

**Cosine of Incident Angle**

# Reflectance Equation




Replace sum with integral

$$L_r(\omega_r) = \int_{\Omega} L_i(\omega_i) f(\omega_i, \omega_r)(\omega_i \cdot n) d\omega_i$$

**Reflected Radiance (Output Image)**

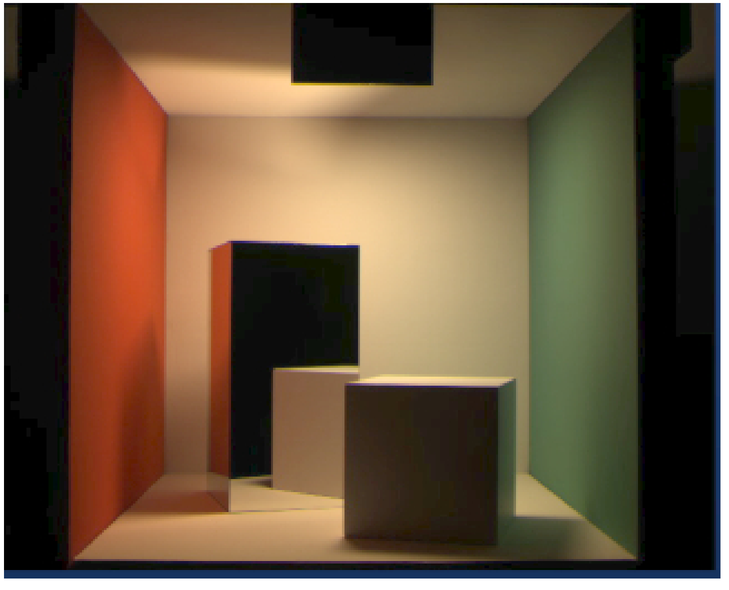
**Incident Radiance (from light soure)**

**BRDF**

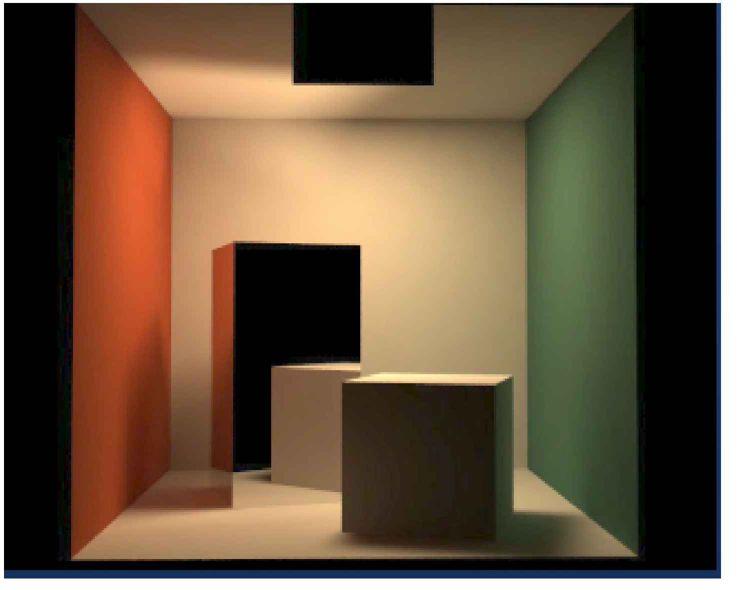
**Cosine of Incident Angle**

# Matching Reality

- Cornell box comparison



**Photo**



**Rendered**

# Summary

- Radiance is a fundamental field quantity that characterize the distribution of light in environment
  - quantity associate with a light ray
  - constant along a ray in vacuum
- We use BRDF to characterize surface material property
- Rendering is about computing reflected radiance
  - The reflectance equation

# Next time…

- We’ll have a midterm review
- The second programming assignment is due on midnight next Tuesday (10/10)
- We’ll have midterm exam on next Thursday (10/12) – worth 15% of your total score
  - Exam will focus on lecture material
  - Question types: mostly explanations, definitions, and some calculations
  - Closed-book, no computer, no smartphone
  - But, you are allowed to bring one single-page letter-sized cheatsheet and calculator