Volume Rendering

Volume Rendering versus Other Techniques

- Volume rendering: creates a 2D image directly from 3D volumetric data
  - Mapping the entire 3D data into a 2D image
  - Volume images retain full information at the cost of increased algorithm complexity and rendering times
  - Rendering of amorphous phenomena such as clouds, fog

- Surface rendering: creates an image of a surface contained within the volume data using geometric primitives
  - Triangles or points

- Clipping: removes a part of a volume to display the rest of the contents
  - Applicable with volume or surface rendering.

Volume Rendering Techniques

- Object-order technique
  - Uses a forward mapping scheme where the volume data is mapped into the image plane

    Splatting

- Image-order technique
  - Uses a backward mapping scheme where rays are cast from each pixel in the image plane through the volume data to determine the pixel value

    Ray casting, Volume buffer

- Hybrid technique
  - Combines the two approaches

    Cell-by-cell V-buffer, Shear-warp

Splatting

What is Splatting?

- Object-order rendering: Forward mapping algorithm that maps data samples onto the image plane

- Splatting considers how a sample can contribute to many points in space, and hence to many pixels in the image plane in terms of a distribution function
  - Distributes energy from input samples into space
  - Calculates an image plane footprint for each data sample and uses the footprint to spread the sample's energy onto the image plane
  - Works for regular grid with/out the same spacings in all directions
  - Can be parallel.

L. Westover, Footprint evaluation for volume rendering, Computer Graphics, vol 24, no. 4, August 1990

Basic Four Steps

- Transforming sample from input <*i,j,k*> grid space to <*x,y,z*> screen space
- Shading sample using some shading rule: <*x,y,z,R,G,B,A*>
- Reconstruction step: Determine the portion of the image the sample can affect and add the sample's contribution to the sheet accumulator
  - Defining and computing footprint function
  - Every data sample is convolved with a reconstruction kernel to produce the image of this data sample.
- Visibility: Matt the sheet accumulator to the working image using a compositing operator when all samples that lie in a sheet are processed
- Once all samples are processed, the working image becomes the final image.

Contribution of a Voxel to Image Plane

- **Consider a volume rendering in which the values of data samples represent density distribution**

$$\rho(s) = \rho(x_s, y_s, z_s)$$

- **Each data sample *s* contributes to many points in space**

$$C_s(x, y, z) = h(x - x_s, y - y_s, z - z_s)\rho(s)$$

where *h()* is the volume reconstruction kernel. One can treat each data sample individually and spreads its contribution to the output samples.

- **Contribution of sample *s* to an image plane pixel *p*→*(x,y)***

$$I_s(p) = I_s(x, y) = \rho(s) \int_{-\infty}^{\infty} h(x - x_s, y - y_s, w)dw$$

where *w* is along a ray through the kernel that is perpendicular to the screen with its origin at <x,y>.

Splat Footprint

- Defining a footprint function

$$F(x,y) = \int_{-\infty}^{\infty} h(x,y,w)dw$$

(x,y) denotes the displacement of an image sample (a pixel) from the center of the sample's image plane projection

- Contribution to the ray color at pixel *p* is

$$I_s(p) = \rho(s)F(x - x_s, y - y_s)$$

F is independent of the sample's density

F works as the weight of a voxel's contribution to the image

Total Contribution

- Ray color for pixel $p \rightarrow (x,y)$ is

$$I(p) = \sum_s I_s(p) = \sum_s \rho(s) F(x - x_s, y - y_s)$$

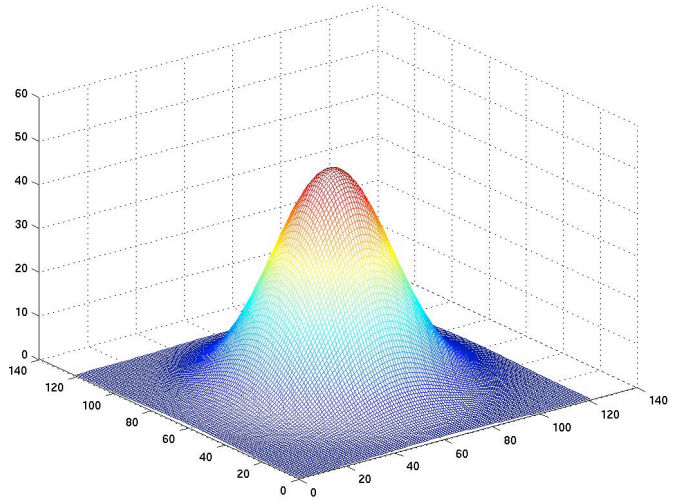
The weight at each pixel is given by footprint F for different sample s

$<x_s,y_s>$ denotes the sample's image plane projection and $<x,y>$ denotes the pixel's image plane location

Reconstruction Kernel

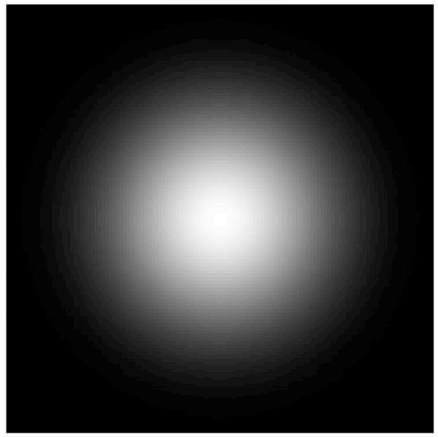
- Splat footprint is integral of reconstruction kernel *h()*

$$F(x,y) = \int_{-\infty}^{\infty} h(x,y,z)dz$$



- Gaussian kernel

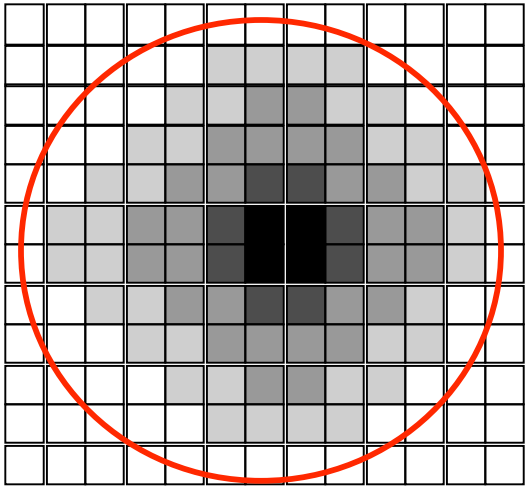
$$h(x,y,z) = \exp(-x^2/a^2 - y^2/b^2 - z^2/c^2)$$



- If Gaussian reconstruction kernel is spherical (a=b=c), footprint is isotropic

Footprint Table

- Footprint can be computed analytically for some reconstruction kernels (such as Gaussian)
- A footprint table can be generated with discrete integration
- For orthographic projection, all voxels have the same footprint, except for an image space offset $(x\text{-}x_s, y\text{-}y_s)$
  - ➢ One footprint table is needed per view, and can be pre-computed and stored in a look-up table
  - ➢ View-transformed footprint table can be created from one generic footprint



A footprint Table

Gray-scale value:
White = zero weight
Black = highest weight
Circle = footprint area

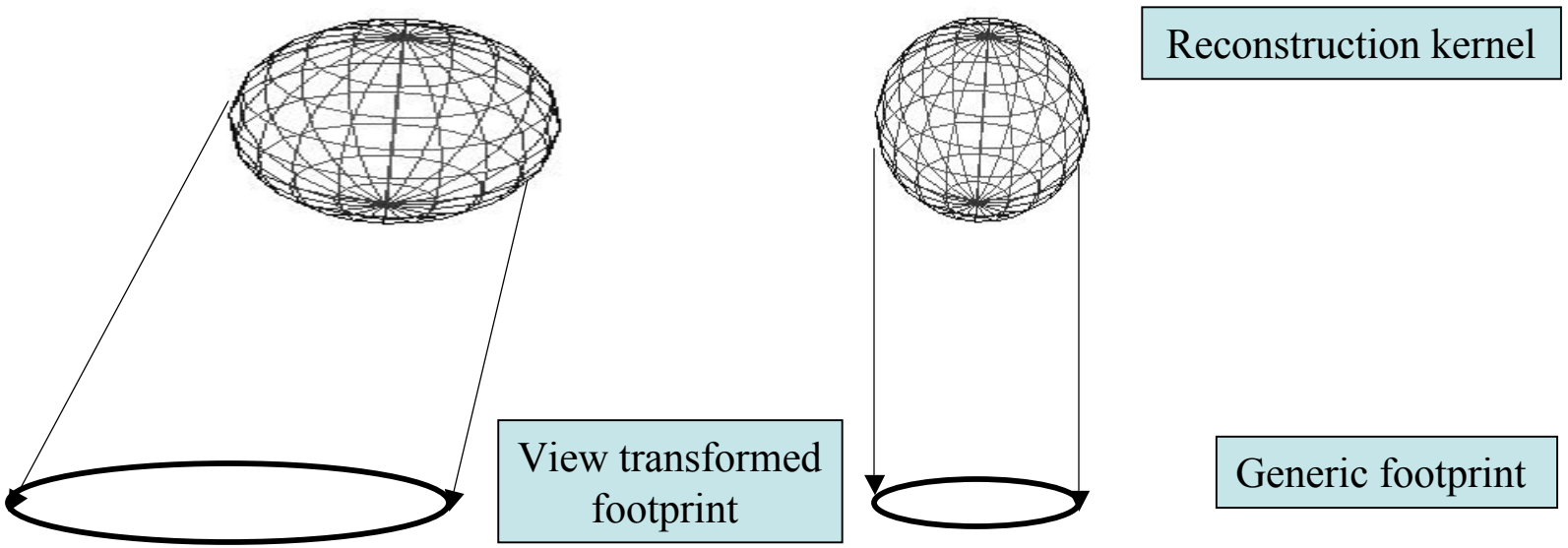
View-Transformed Footprint Table

- Calculate the screen space extent of the projection of the kernel
- Map the view-transformed extent to an extent that surrounds the projection of a generic kernel (pre-computed)
- Use the view-transformed table for each sample by centering the table at each sample's projected screen position
- Generic kernel is a sphere:
  - Two basic cases may result for extents and mapping
    Sphere and ellipsoid

Spherical and Elliptical Kernels

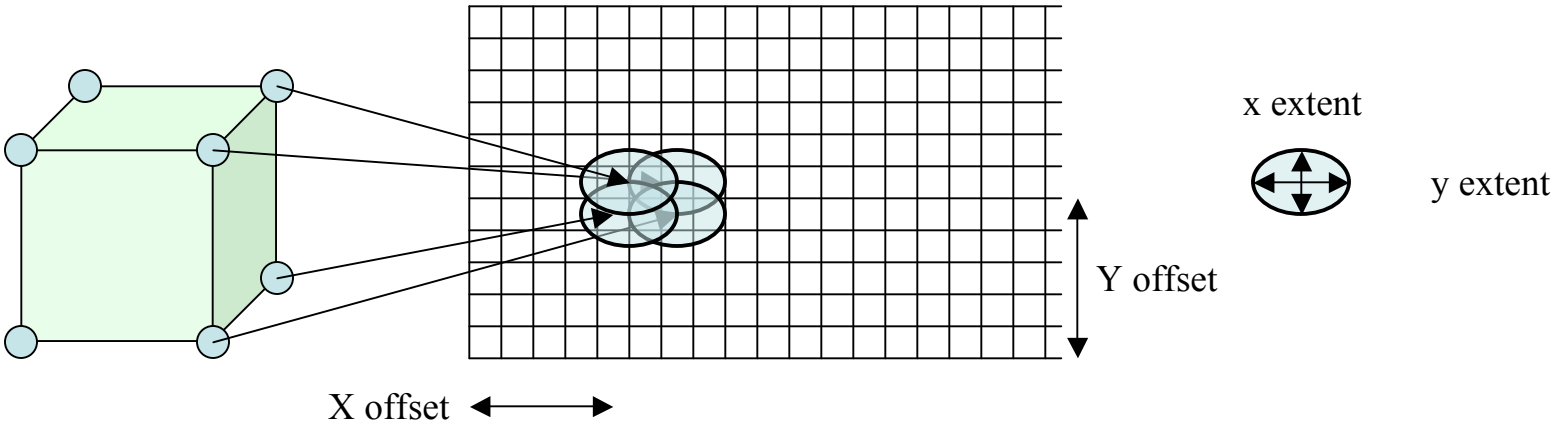
- A sphere results when the input volume has equal spacings along all grid directions and the viewing transform has only uniform scaling. The sphere maps to circular footprint function.

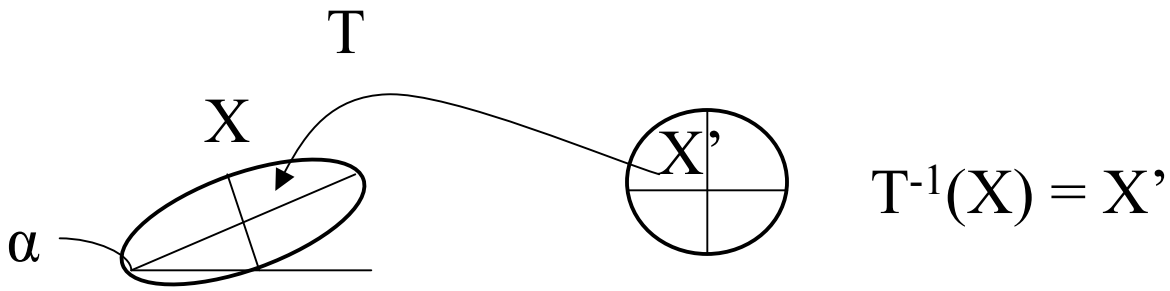
- An ellipsoid is more general then the spherical case. It results if there is a difference in the scaling factors between the axis. This ellipsoid maps to an elliptic footprint function.



Extent and Mapping

- Calculate the offset and extent of each footprint.
- T^{-1} is the mapping that for each point X in the view-transformed footprint finds the point X' in the generic footprint.





The Graphics Pipeline

Generic footprint kernel (preprocess stage)

Find offset of the data point into the screen

Find all pixels in the extent of that footprint

For every pixel get the reverse mapping into the generic table

Accumulate the contributions to every pixel

Subsequent Developments

- The basic splatting algorithm was refined by more recent work.
- K. Mueller et al., High quality splatting on rectangular grids with efficient culling of occluded voxels – *IEEE Trans. on Visualization and Computer Graphics*, 1999.
- J. Huang et al. FastSplats: Optimized splatting on rectilinear grids, FastSplats: Optimized splatting on rectilinear grids, *IEEE Visualization'00*, Salt-Lake City, October 2000
- N. Neophytou and K. Mueller, Space-time points: 4D Splatting on efficient grids, *Symposium on Volume Visualization and Graphics 2002*, Boston, October 2002
- N. Neophytou and K. Mueller, Post-convolved splatting, *Joint Eurographics - IEEE TCVG Symposium on Visualization* 2003, Grenoble, France, May 2003

Examples

- J. Huang et al. FastSplats: Optimized splatting on rectilinear grids, 2000.