Vector Visualization

Vector data

- A vector is an object with direction and length

$$\boldsymbol{v} = (v_x, v_y, v_z)$$

- A vector field is a field which associates a vector with each point in space
- The vector data is 3D representation of direction and magnitude
- Examples include

  Fluid flow, velocity $\boldsymbol{v}$

  Electromagnetic field: $\boldsymbol{E}, \boldsymbol{B}$

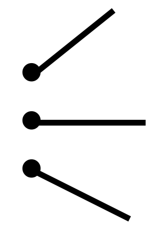
  Gradient of any scalar field: $\boldsymbol{A} = \nabla T$

Different Visualization Techniques

- Hedgehogs
  - Point icons
- Advection based methods
  - Lines, surfaces, tubes
- Line integral convolution
- Global technique
  - Critical points

Point Icons

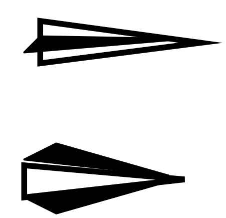
- Draw point icons at selected points of the vector field



- A natural approach is to draw an oriented, scaled line for each vector $\boldsymbol{v} = (v_x, v_y, v_z)$
  - The line begins at the point with which the vector is associated and is oriented in the direction of the vector components

  Arrows are added to indicate the direction of the line

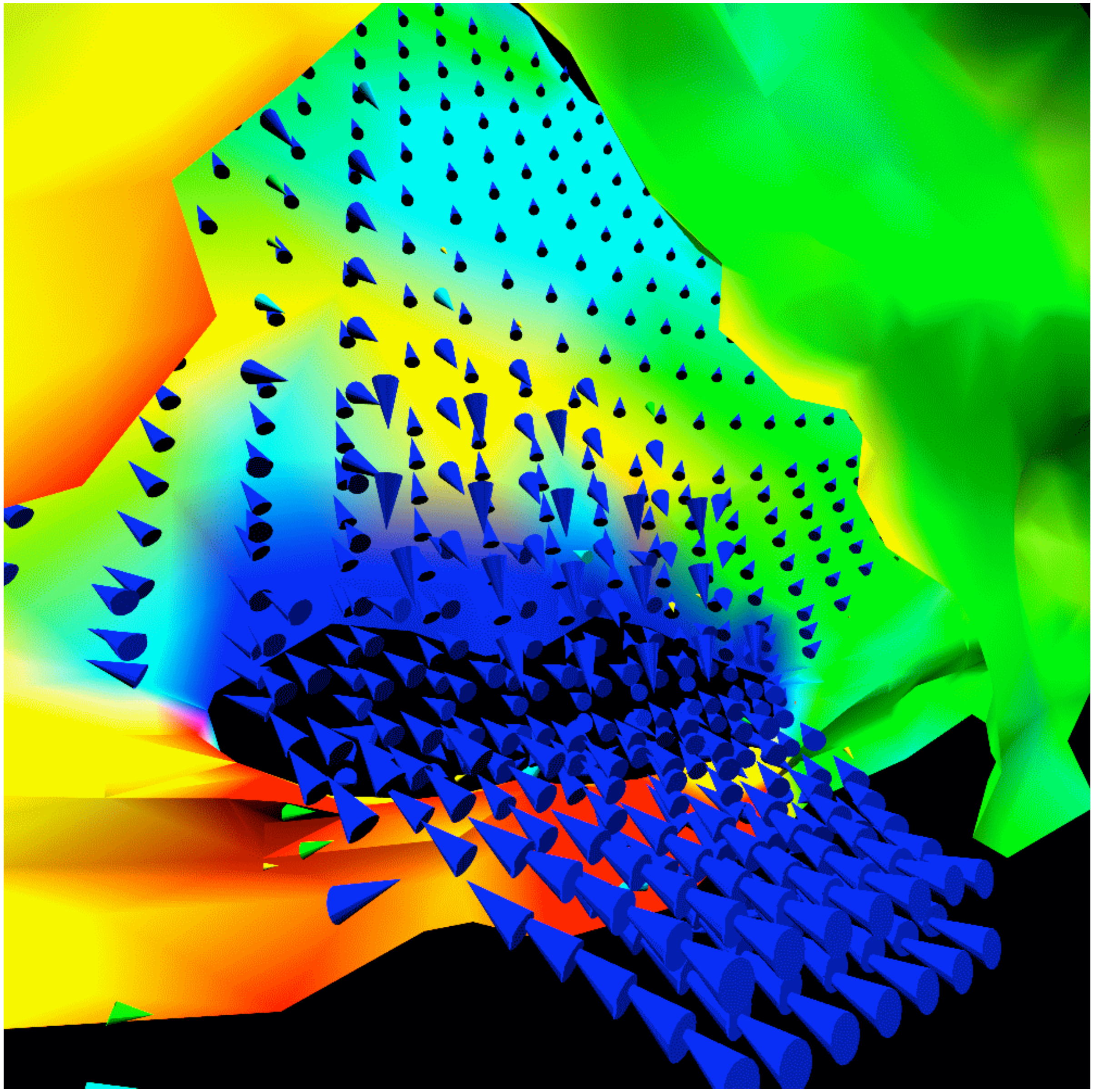
  Lines may be colored according to the vector magnitude or type of the quantity (e.g. temperature)



- Oriented glyphs (such as triangle or cone) can be used

- To avoid visual clutter, the density of displayed icons must be kept very low
  - Point icons are useful in visualizing 2D slices of 3D vector fields

Air Flow over Windshield



Air flow coming from a dashboard vent and striking the windshield of an automobile

http://www-fp.mcs.anl.gov/fl

Line Icons

- Line icons provide a continuous representation of the vector field, thus avoiding mental interpolation of point icons

- Line icons are particles traces, streaklines, and streamlines

- In general, these three families of trajectories are distinct from each other
  - turbulent flows where flow pattern is time dependent

- But they are equivalent to each other in steady flows

Particle Traces or Pathlines

- are trajectories traversed by fluid particles over time
  - An animation over time can give an illusion of motion
- are computed by integrating $\frac{d\vec{x}}{dt} = \vec{v}(\vec{x},t)$

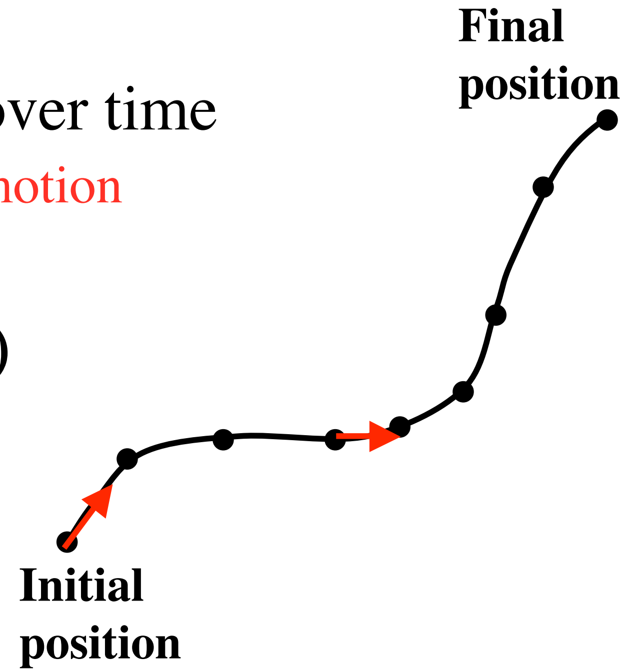
  with initial condition $\boldsymbol{x(0) = x_0}$

  $\boldsymbol{v(x,t)}$ is a vector field where $\boldsymbol{x}$ is the position in space and $\boldsymbol{t}$ is time
  - Accuracy of numerical integration is crucial

  $$\vec{x}_{i+1} = \vec{x}_i + \vec{v}_i \Delta t$$

  simplest form

  Runge-Kutta of higher order can be used
- give a sense of complete time evolution of the flow
- are experimentally visualized by injecting instantaneously a dye or smoke in the flow and taking a long exposure photograph



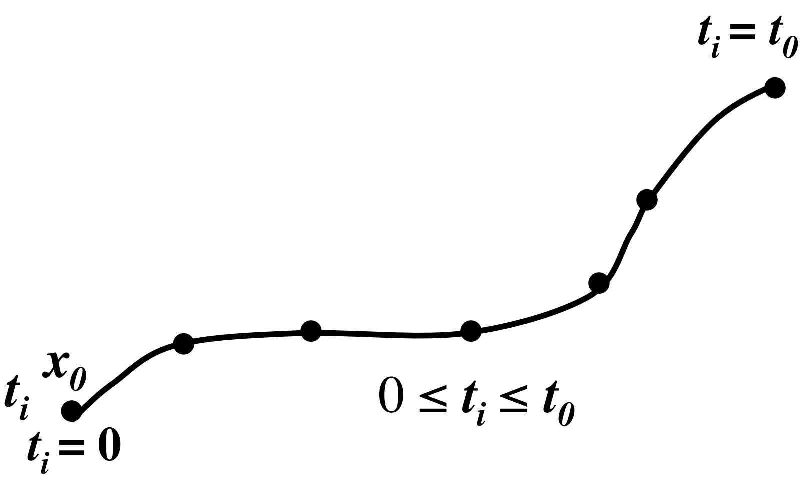
Streaklines

- A streakline is the locus at time t_0 of all the fluid particles that have previously passed through a specified point x_0
  - ➢ Line connecting particles released from the same location

- is computed by linking the endpoints of all trajectories between times t_i and t_0 for every t_i such that $0 \le t_i \le t_0$

- gives information on the past history of the flow

- is experimentally observed by injecting continuously at the point x_0 a tracer such as hydrogen bubbles and by taking a short exposure photograph at time t_0



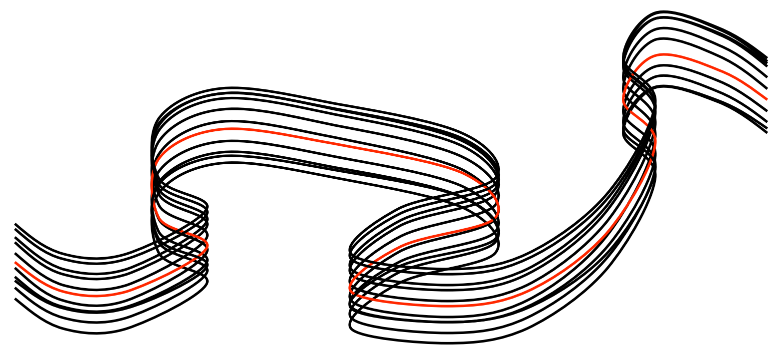
All particles passed through point x_0 in the field

Streamlines or Fieldlines

- are integral curves satisfying

$$\frac{d\vec{x}}{ds} = \vec{v}(\vec{x}, t_0)$$

where ***s*** is a parameter measuring the distance along the path



- are everywhere tangent to the steady flow

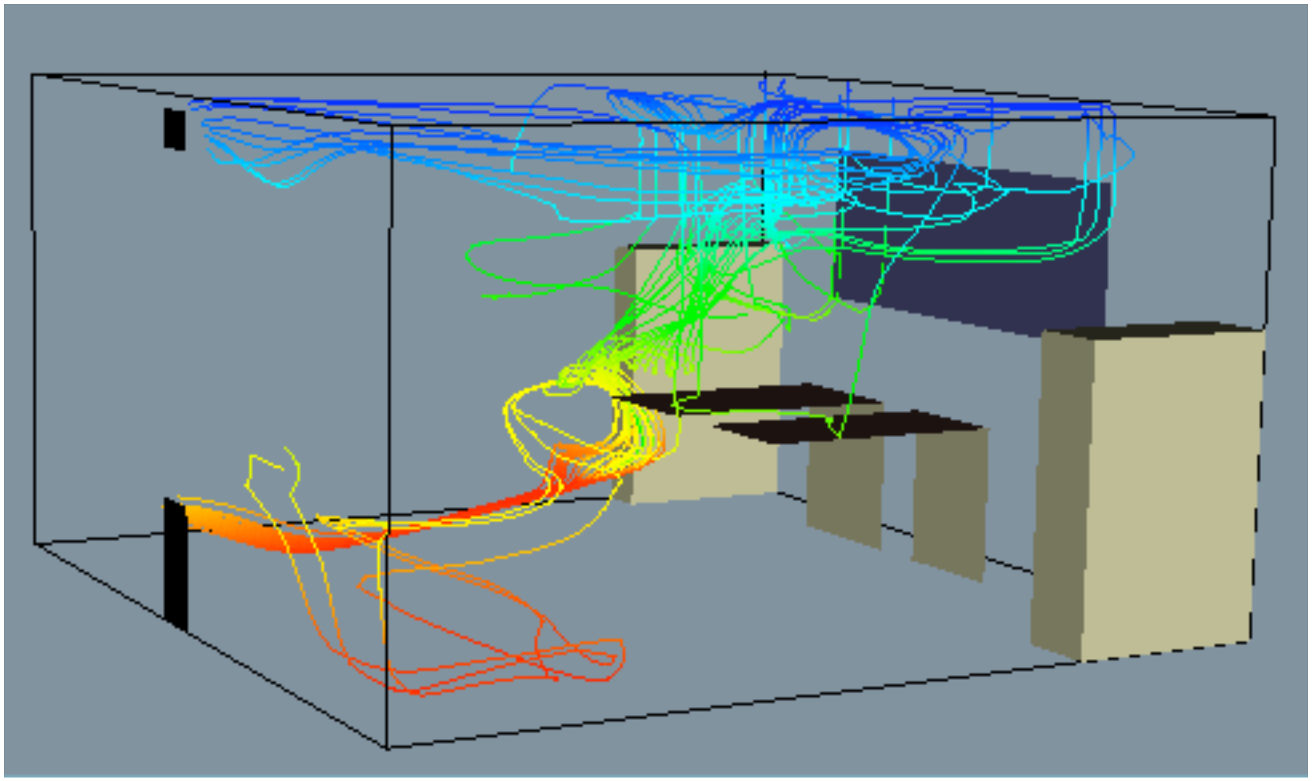
- provide an instantaneous picture of the flow at time t_0

curve ***s(x,t)***

- are experimentally visualized by injecting a large number of tracer particles in the flow and taking a short-time exposure photograph at t_0

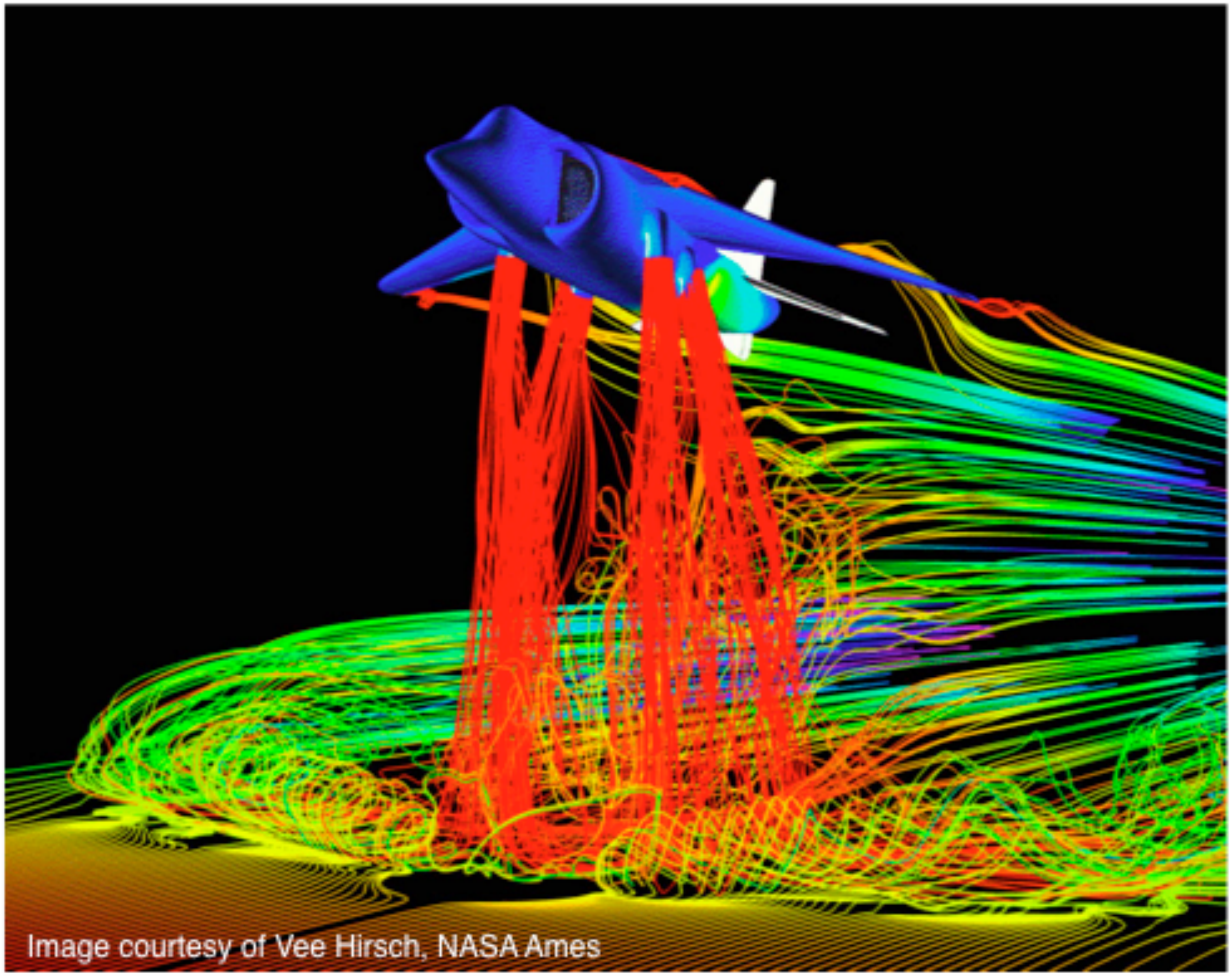
$$s = \int_t \vec{v} ds$$

Streamlines through a Room



Streamlines are color coded to display pressure variation

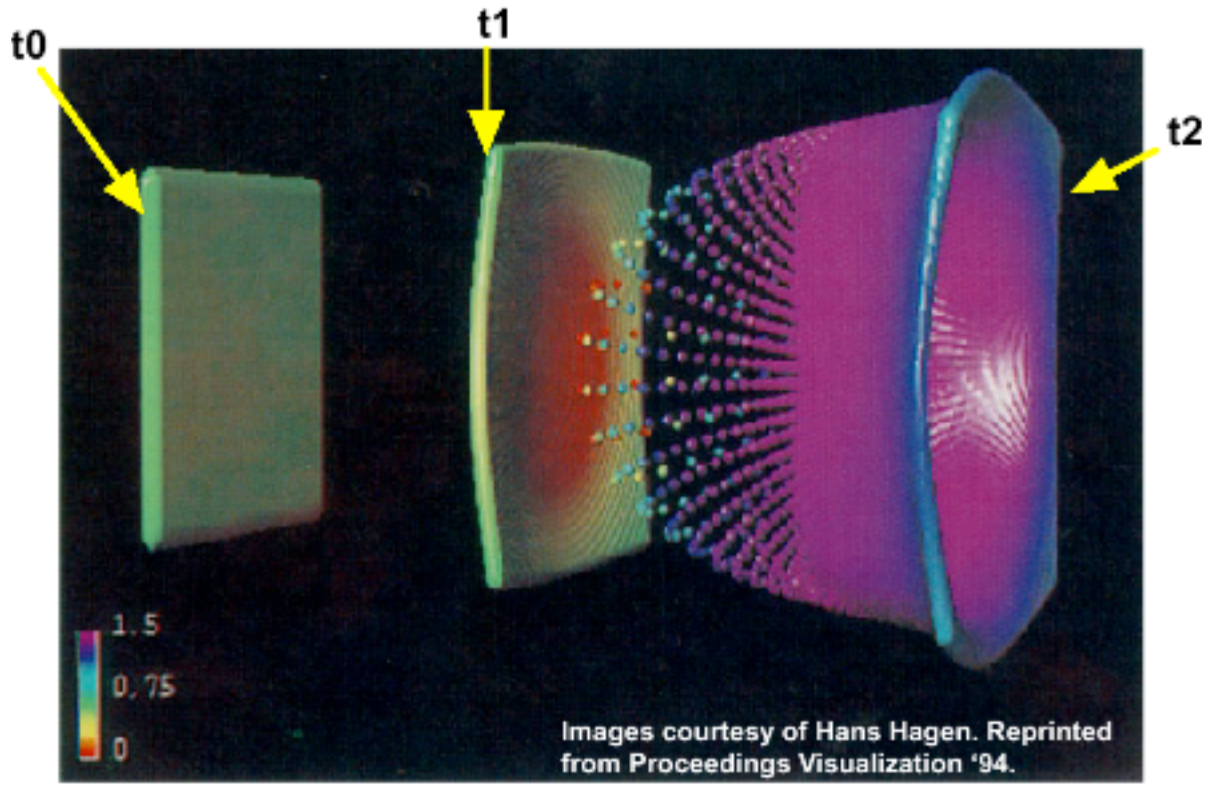
Streamlines from Plane



Many streamlines can give an overall feel for the vector field

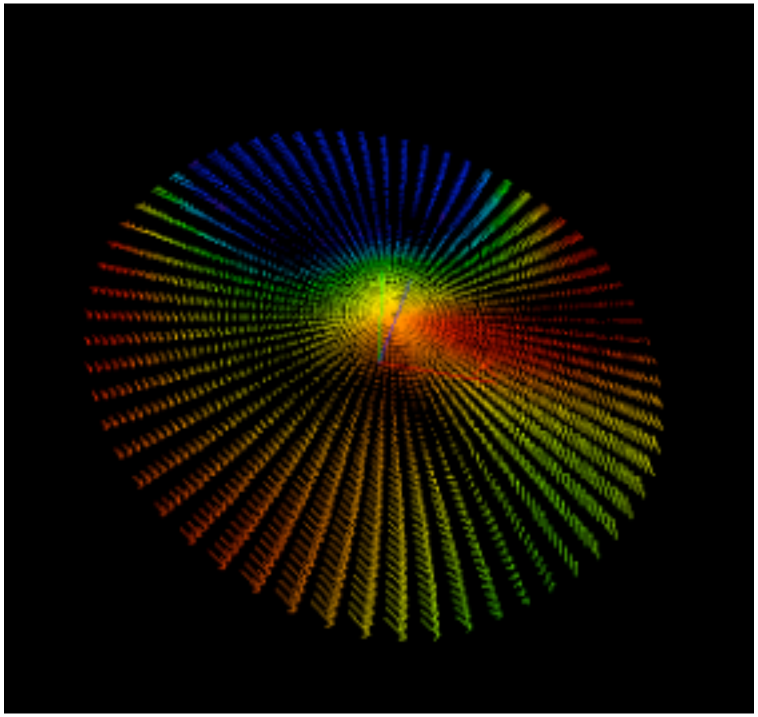
Timelines

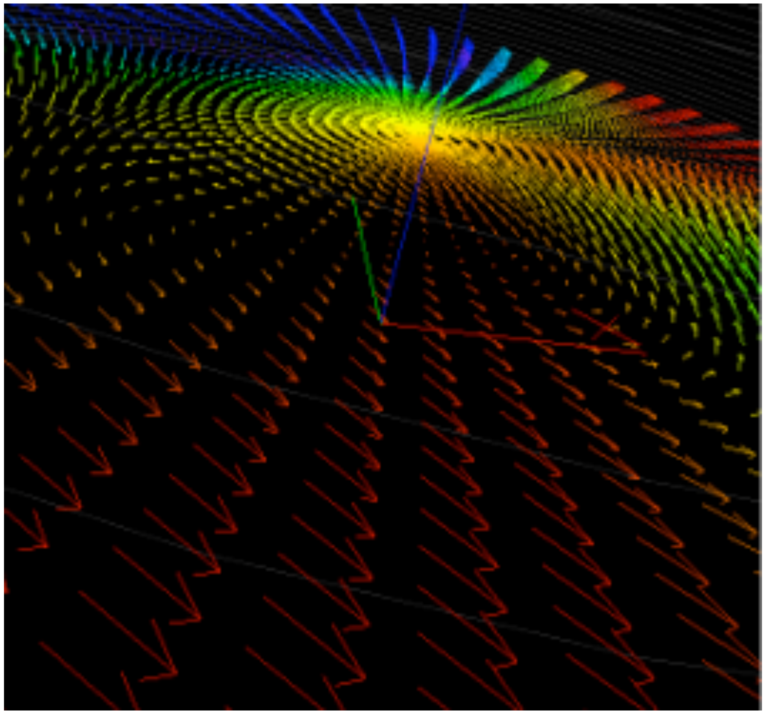
Are lines connecting particles released simultaneously



Flow in Eye Anterior Chamber

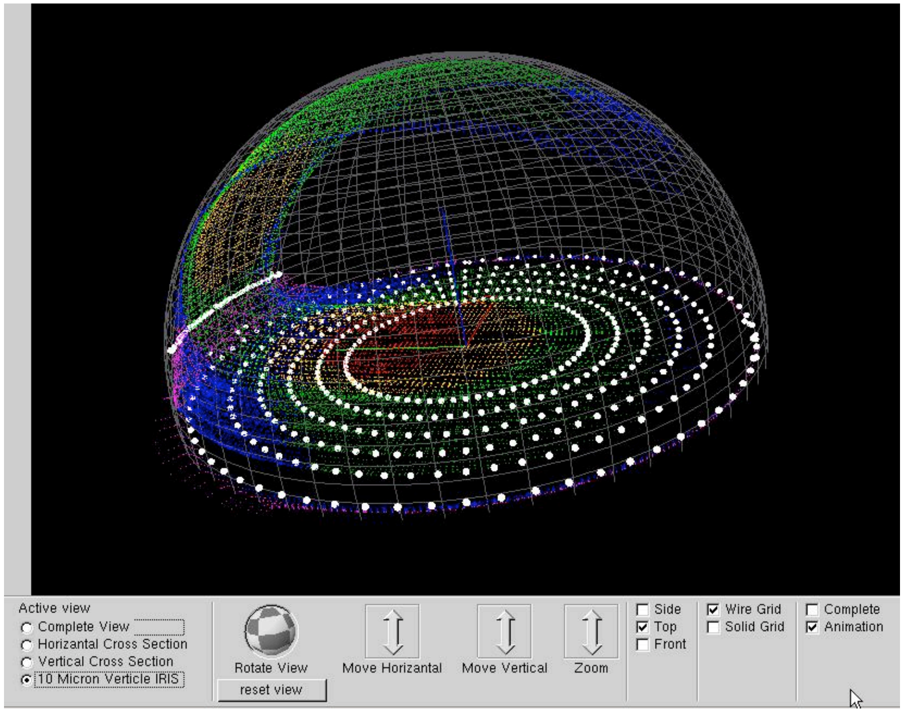
- Experimental data for velocities of particles at 27 positions in the anterior chamber of the rabbit eye.
- More datasets are generated by interpolation/extrapolation.
- Velocities displayed with color-coded arrows (to indicate depth).

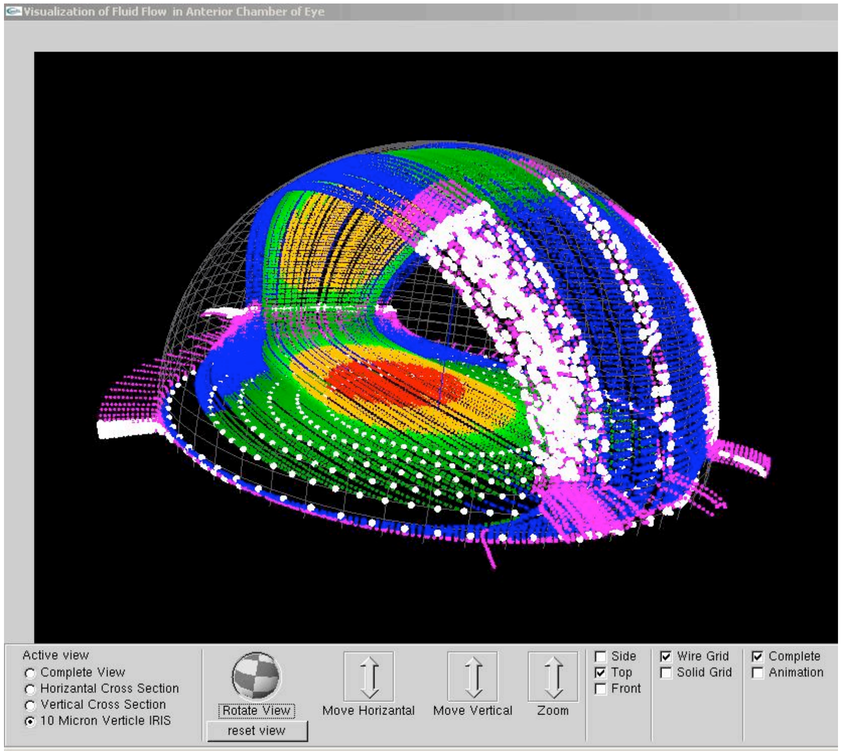




Animation of Flow Particles

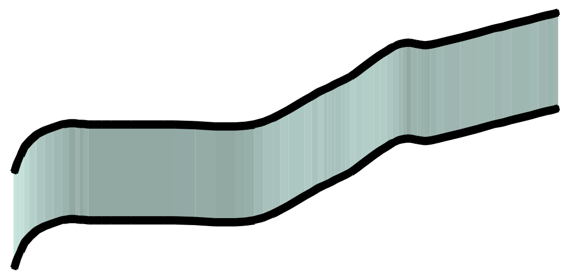
- Particles are initially arranged in a number of concentric rings
- As time evolves, particles move with different velocities as determined by Computational Fluid Dynamics (CFD) simulation.
- Color-coding to indicate the magnitude of flow particle velocities
  - Red for high, green for average, blue for low and white for almost zero





Streamribbons

- Are narrow surfaces defined by two adjacent streamlines and then bridging the lines with a polygonal mesh



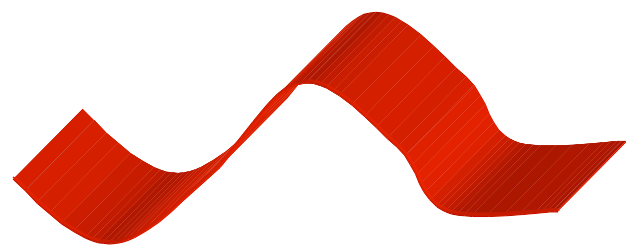
- Provide information about two flow parameters

  Vorticity (***w***)- a measure of rotation of the vector field

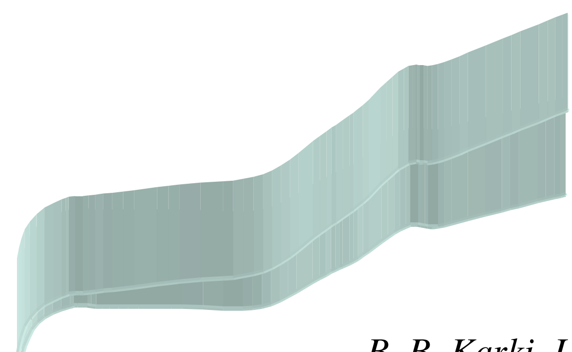
  - Streamwise vorticity (Ω) is rotation of the vector field around the streamline
  - Twist rate encodes the streamwise vorticity

  Divergence - a measure of the spread of the flow

  - Changing width is proportional to the cross-flow divergence

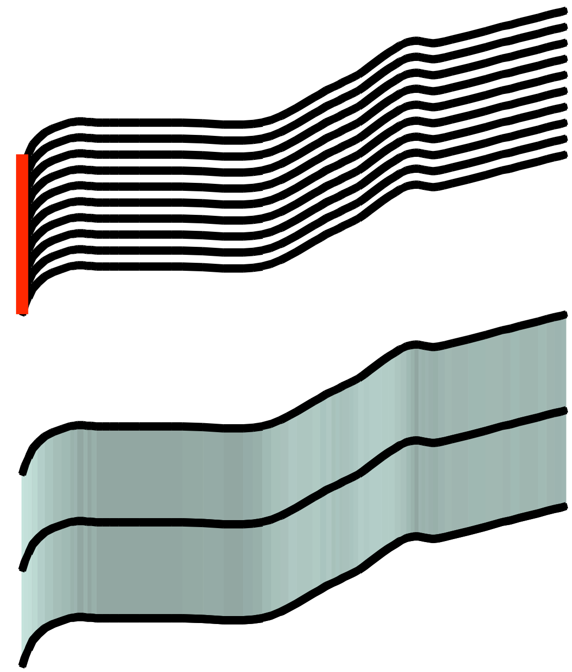


- Adjacent streamlines in divergent flows tend to spread; and one can adaptively adds particles to the advecting front.



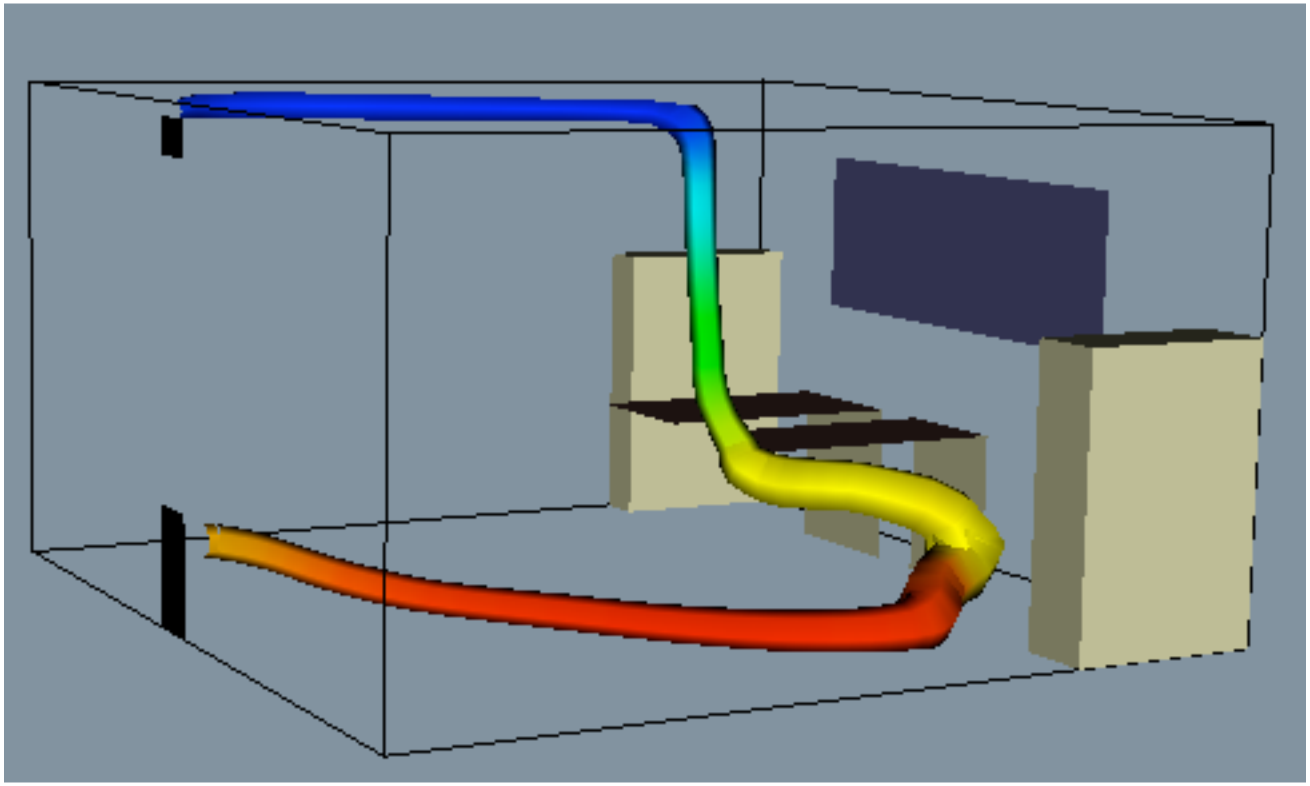
Streamsurfaces

- A streamsurface is a collection of an infinite number of streamlines passing through a base curve
  - ➢ advecting a material line segment, i.e., a front of particles
  - ➢ Base curve or rake defines the starting points for the streamlines
  - ➢ Any point on the streamsurface is tangential to the velocity vector
  - ➢ Streamsurfaces is a collection of streamribbons connected along their edges



- If the base curve is closed (e.g., a circle), the surface is closed and a streamtube results
- No fluid can pass through the surface
  - ➢ Streamtubes are then representation of constant mass flux
- No polygonal tiling: a large number of surface particles gives appearance of a continuous surface

Streamtube Through a Room



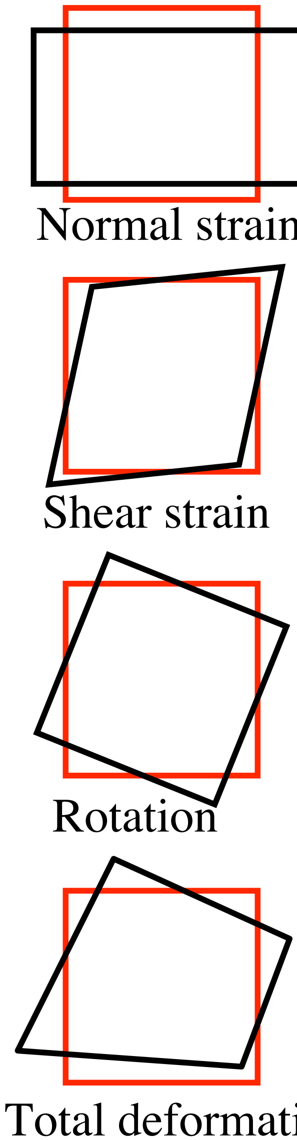
A single streamtube with diameter of the tube indicating flow-velocity, and color representing pressure variation. A thinner tube indicates faster flow.

Streampolygons

- To visualize local properties of strain, displacement, and rotation besides streamwise vorticity and divergence

- Nonuniform vector fields give rise to local deformation in the region where they occur
  - Deformation consists of local strain (normal and shear distortions) and rigid body rotation

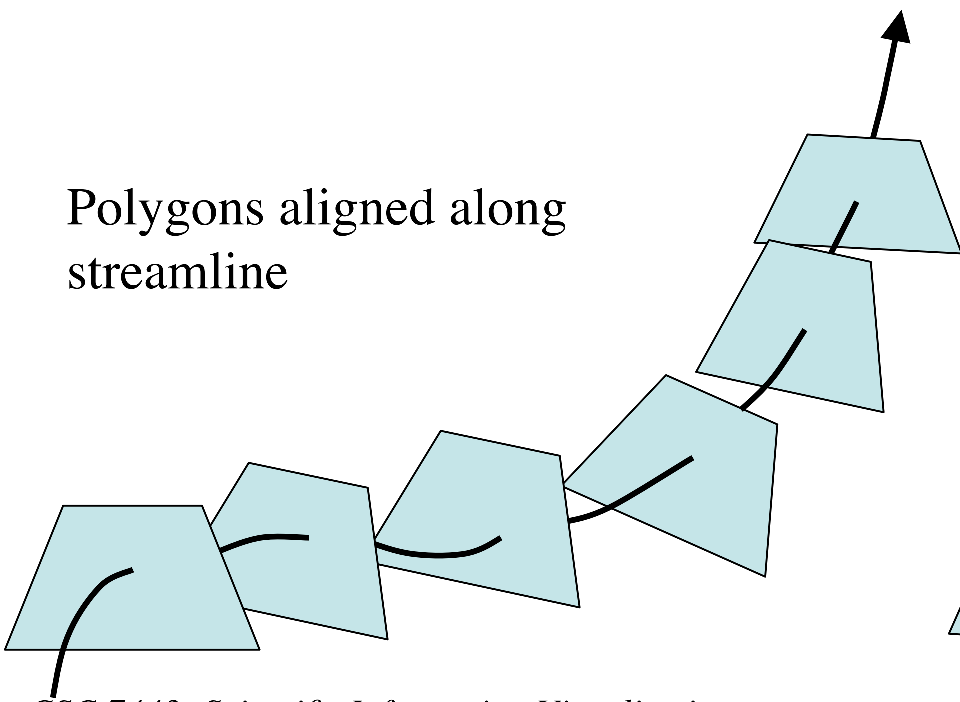
- A polygon is placed into a vector field at a specified point and then deformed according to local strain
  - Align the polygon normal with the local vector
  - Rigid body rotation about the local vector is thus the streamwise vorticity
  - Effects of strain are in the plane perpendicular to a streamline passing through the point

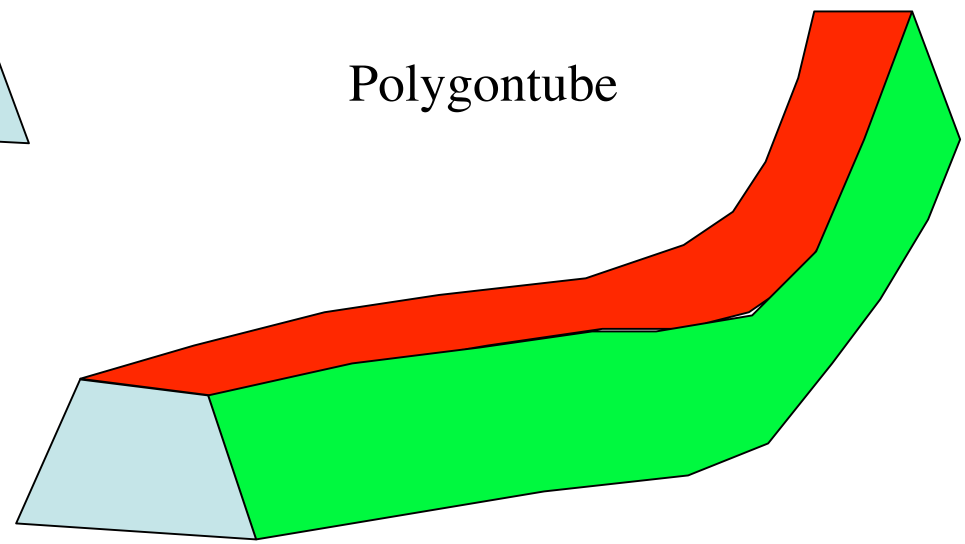


Streamtube from Polygons

- Stream polygon may be swept along a streamline to generate tube
  - Radius of the tube can be varied according to the scalar function vector magnitude

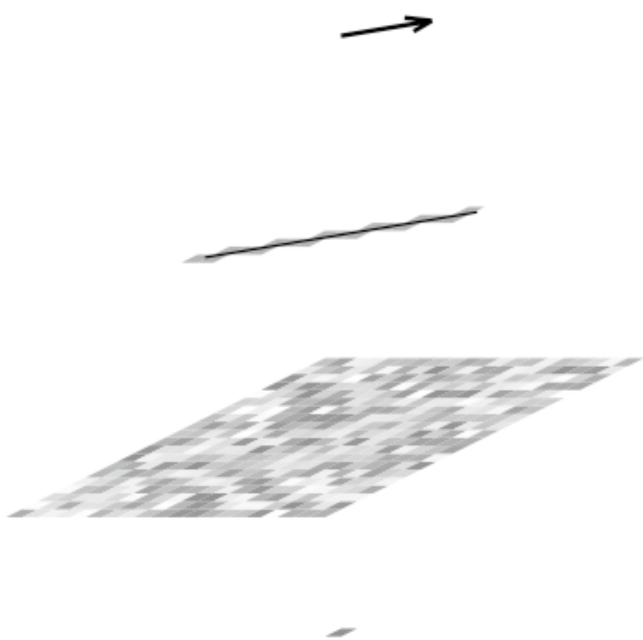
$$r(\vec{v}) = r_{max}\sqrt{\frac{|\vec{v}_{min}|}{|\vec{v}|}}$$

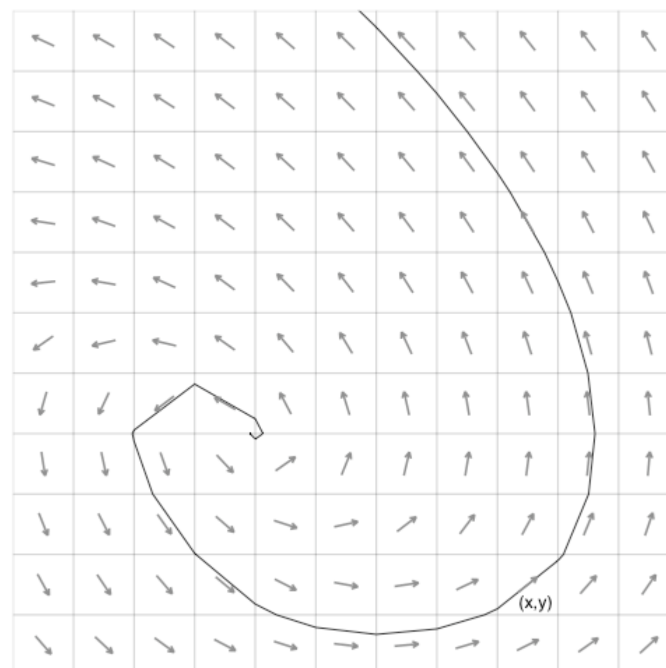




Line Integral Convolution (LIC)

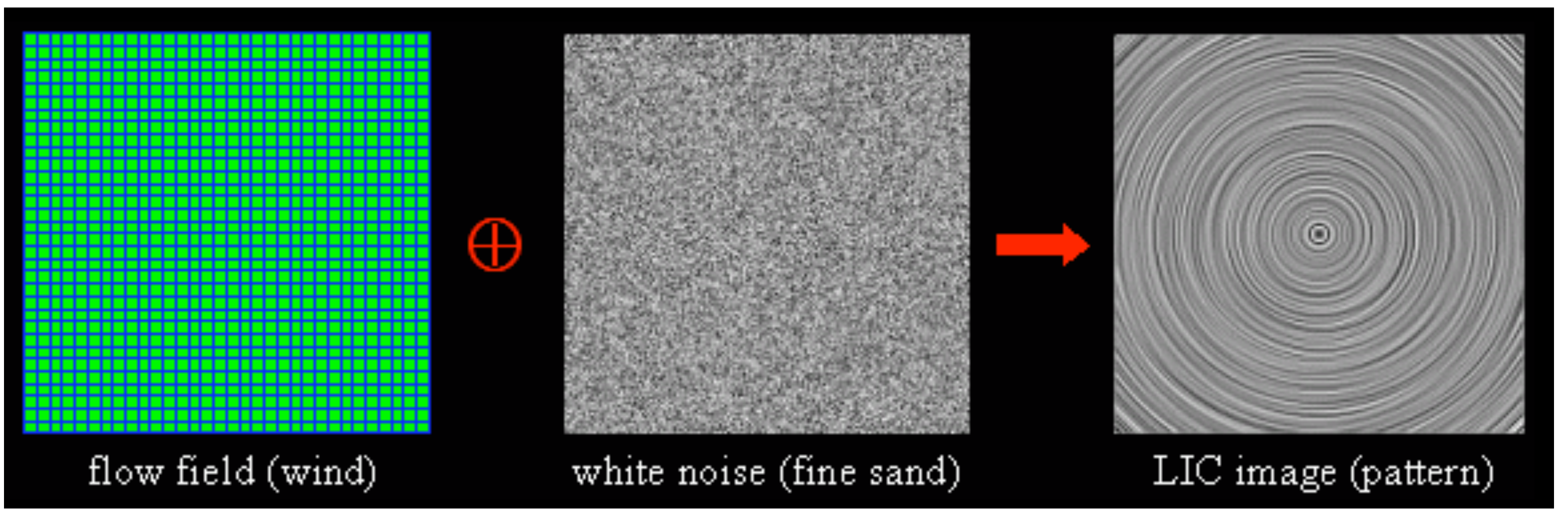
- Input: Vector field and texture image
  Output: Colored field correlated along in the flow direction
- Texture image is normally a white noise.
- For each pixel, generate a streamline both forwards and backwards of fixed length
  Integrate the intensity that streamline passes through



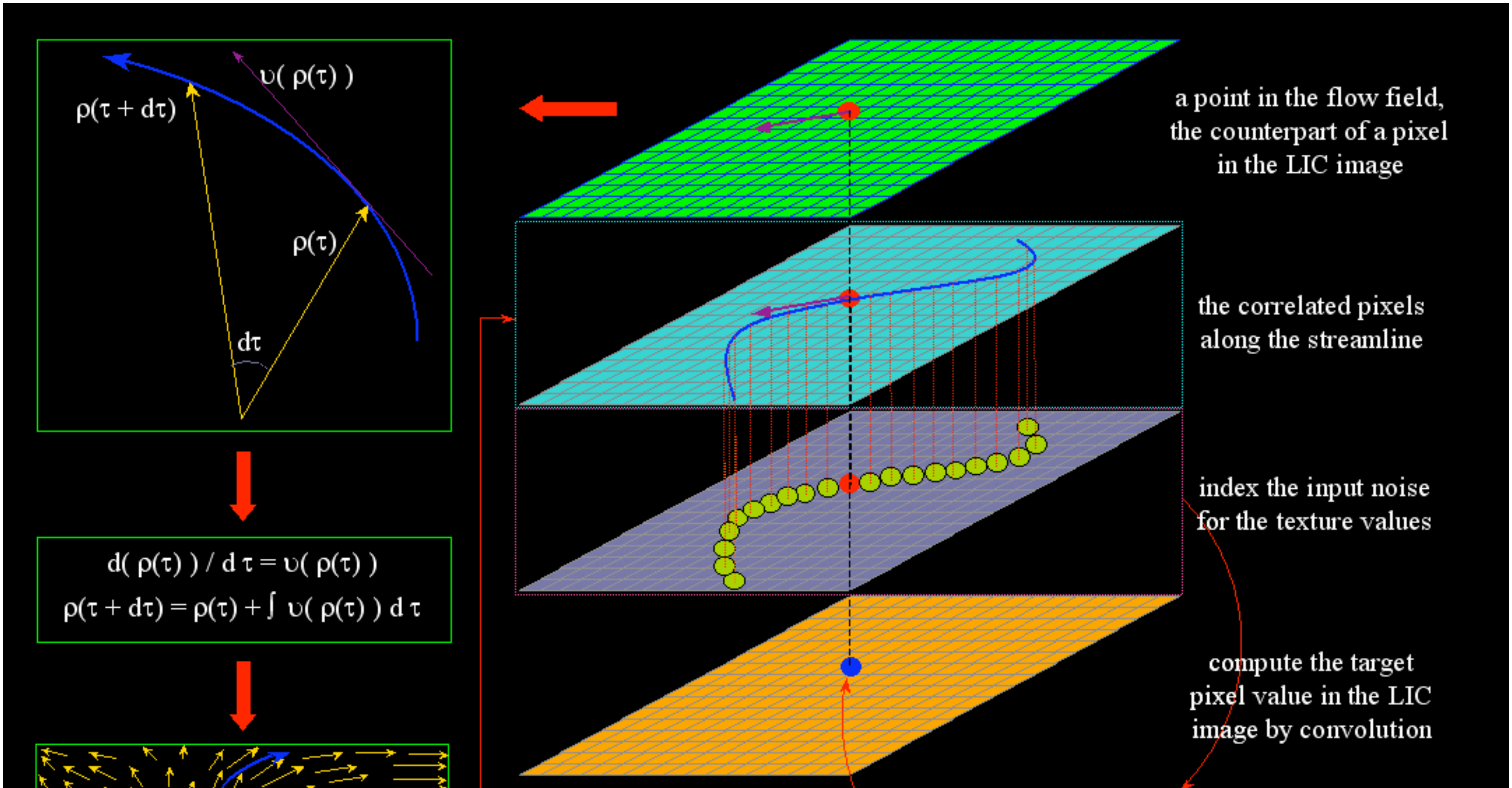


LIC Image

- LIC emulates the effect of a strong wind blowing across a fine sand



LIC: Basic Idea



$$T'(\rho(0)) = \frac{\int_{-L}^{L} K(\tau)\, T(\rho(\tau))\, d\tau}{\int_{-L}^{L} K(\tau)\, d\tau}$$

LIC - Example

Flowers processed using LIC with L equal to 0, 5, 10 and 20 (left to right, top to bottom).

Vector Field Topology

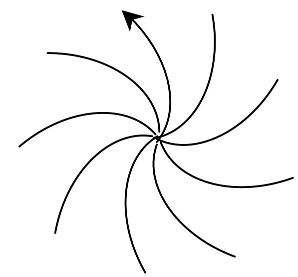
- Complex structures with critical points in the vector field

  Critical points are locations where the local vector magnitude goes to zero and the vector direction becomes undefined

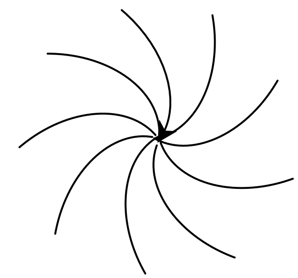
- Vector field $\vec{v} = (v_1, v_2, v_3)$ near a critical point is characterized by the partial derivatives that form a 3x3 Jacobian matrix J

$$v_i = J_{ij} dx_j \qquad J_{ij} = \frac{\partial v_i}{\partial x_j}$$

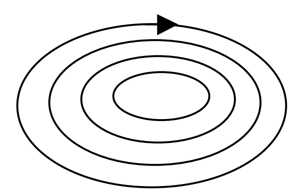
- The eigenvalues of J consists of real and imaginary components
  - Real part describes the relative attraction or repulsion of the vector field to the critical point
  - Imaginary part describes the rotation of the vector field around the critical point



Repelling Focus



Attracting Focus



Center

Visualizing Global Structure

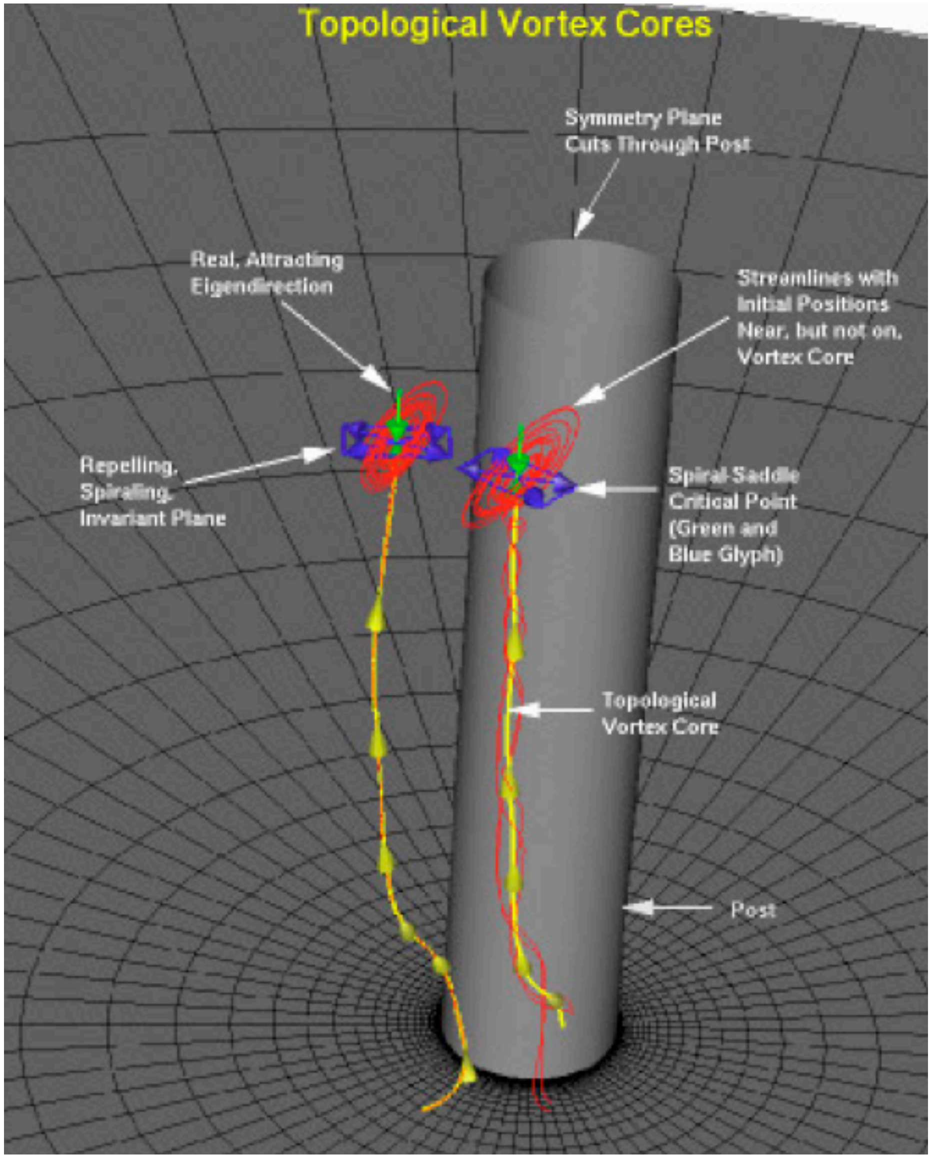
- Provide a global understanding of the field displaying critical points with glyphs (local field pattern)

- Streamlines will begin or end at the points of detachment or attachment

- Helicity-density (H_d) shown using isosurfaces gives an indication for the location and structure of a vortex
  - H_d is a scalar function of the dot product between vortcity (w) and local vecor (v)

$$H_d = \vec{v}.\vec{w} = |\vec{v}||\vec{w}|\cos\varphi$$

  - Large +ve values of H_d: right-handed vertices
  - Large -ve values of H_d: left-handed vertices



Detection of Vortices

- Swirl parameter method
  - Connection between swirling motion and existence of complex eignevalues in the velocity gradient tensor *J*

    Swirl parameter: $$\tau = \frac{t_{conv}}{t_{orbit}} = \frac{|\mathrm{Im}(\lambda_C)|L}{2\pi|v_{conv}|}$$

    Convection time = the time for a particle to convect through the region of complex eigenvalues R_C

    Orbit time = the time for a particle to return to the same angular position (L is the characteristic length).
  - Non-zero value of swirl parameter in the regions containing vortices.

- Lambda$_2$ method
  - Form a real symmetric tensor = $S^2 + \Omega^2$.

    S and Ω are symmetric and antisymmetric parts of *J*.
  - A vortex is a region where $S^2 + \Omega^2$ has two negative values:

    A negative λ_2 means that the corresponding point belongs to a vertex core.

More on Vortices

- Eigenvector method
  - Based on critical-point theory:

    Swirling flows do not always need to contain critical points within their centers
  - One real eigenvalue and other two complex eigenvalues

    If velocity vectors projected onto the plane normal to the real eigenvector are zero, then the point must be part of the vortex core.

- Streamline method
  - Winding angle:

    Measures the amount of rotation of the streamline with respect to a point.

    Gives the cumulative change of direction of streamline segments:

$$\alpha_w = \sum_{i=1}^{N-2} \angle(p_{i-1}, p_i, p_{i+1})$$

  - A vortex exists in a region where $\alpha_\omega \geq 2\pi$ for at least one streamline.

References

- Carbal, Imaging vector fields using line integral convolution, Proceedings of ACM SigGraph 93, 263, 1993.
- Computer Visualization: Graphics techniques for Scientific and Engineering Analysis by R. S. Gallagher, 1994
- The Visualization Toolkit: An object-oriented approach to 3D graphics by W. Schroeder et al., 1997
- www.erc.msstate.edu/~zhanping/Research/FlowVis/LIC