PRESBURGER ARIHMETIC AND FSA

FSA for Solutions of Linear Integer Inequalities:

- Example: $x + 3y \geq 10$ or $3x - 2y \geq 10$.
- $3x - 2y = 10$ is equivalent to 2 inequalities:

$$3x - 2y \geq 10 \text{ and } 3x - 2y \leq 10 \text{ (or, } -3x + 2y \geq 10)$$

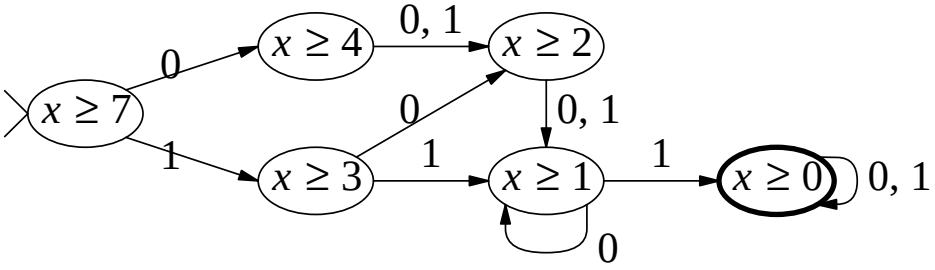
- All variable are non-negative integers.
- Input alphabet = $\{b_0, b_1, b_2, b_3\}$ for inequalities of two variables, and a value-pair $(x, y) = (17, 5)$ is encoded as shown below, with low-order bits represented on the right in the input string.

$$\binom{17}{5} = \binom{1}{0}\binom{0}{1}\binom{0}{0}\binom{1}{1} = b_2b_1b_0b_3; \text{ input} = b_3b_0b_1b_2.$$

- The FSA should accept exactly the string which represent solutions for the inequalities.

Example. We show the method for finding an FSA for solutions of $3x + 5 \geq 24$, i.e., $x \geq 7$. The method generalizes direclty to multiple inequalities involving multiple variables.

Processing rightmost bit of x		New inequality
$x = x'0$;	$x = 2x'$	$2x' \geq 7$, i.e., $x' \geq 4$
$x = x'1$;	$x = 2x' + 1$	$2x' + 1 \geq 6$, i.e., $x' \geq 3$



Question: Show the FSA for $x + 3y \geq 10$ and $3x - 2y \geq 2$.