LIST OF FIGURES

1 Figure 1-1:	Introduction to Multi-Criteria Decision Making
Figure 1-2:	A Taxonomy of MCDM methods (according to
1 18000 1 2.	Chen and Hwang [1991])
2	Multi-Criteria Decision Making Methods5
3	Quantification of Qualitative Data for
	MCDM Problems 23
Figure 3-1:	Actual Comparison Values
Figure 3-2:	Maximum, Average, and Minimum CI Values of
	Random CDP Matrices When the Original
	Saaty Scale is used42
Figure 3-3:	Inversion Rates for Different Scales and Size
	of Set (Class 1 Scales)46
Figure 3-4:	Indiscrimination Rates for Different Scales
	and Size of Set (Class 1 Scales)47
Figure 3-5:	Inversion Rates for Different Scales and Size
	of Set (Class 2 Scales)
Figure 3-6:	Indiscrimination Rates for Different Scales
	and Size of Set (Class 2 Scales)
Figure 3-7:	The Best Scales
Figure 3-8:	The Worst Scales
4	Deriving Relative Weights from Ratio Comparisons 57
Figure 4-1:	Average Residual and CI versus Order of Set
0	When the Human Rationality Assumption is Used
	(the Results Correspond to 100 Random Observations)70
Figure 4-2:	Average Residual and CI versus Order of Set
0	When the Eigenvalue Method is Used
	(the Results Correspond to 100 Random Observations)71
5	Deriving Relative Weights from Difference
	Comparisons73

xiv	MCDM Methods: A Comparative Study, by E. Triantaphyllou
6	A Decomposition Approach for Evaluating Relative Weights Derived from Comparisons 87
Figure 6-1:	Partitioning of the $n(n-1)/2$ Pairwise
Figure 6-2:	Error Rates Under the LP Approach for Sets
E :	Available Comparisons
Figure 0-5:	of Different Size as a Function of the
Figure 6-4:	Available Comparisons
	of Different Size as a Function of the Common Comparisons
Figure 6-5:	Error Rates Under the Non-LP Approach for Sets of Different Size as a Function of the
Figure 6-6:	Common Comparisons
Figure 6-7:	Function of the Available Comparisons
	Function of the Common Comparisons111
7	Reduction of Pairwise Comparisons Via a
7	Reduction of Pairwise Comparisons Via a Duality Approach115
7 Figure 7-1:	Reduction of Pairwise Comparisons Via a Duality Approach
7 Figure 7-1:	Reduction of Pairwise Comparisons Via a Duality Approach
7 Figure 7-1:	Reduction of Pairwise Comparisons Via aDuality Approach
7 Figure 7-1: Figure 7-2:	Reduction of Pairwise Comparisons Via a Duality Approach
7 Figure 7-1: Figure 7-2:	Reduction of Pairwise Comparisons Via aDuality Approach
7 Figure 7-1: Figure 7-2:	Reduction of Pairwise Comparisons Via aDuality Approach
7 Figure 7-1: Figure 7-2: Figure 7-3:	Reduction of Pairwise Comparisons Via a Duality Approach
7 Figure 7-1: Figure 7-2: Figure 7-3:	Reduction of Pairwise Comparisons Via aDuality Approach
7 Figure 7-1: Figure 7-2: Figure 7-3:	Reduction of Pairwise Comparisons Via a Duality Approach
7 Figure 7-1: Figure 7-2: Figure 7-3: Figure 7-4:	Reduction of Pairwise Comparisons Via aDuality Approach
7 Figure 7-1: Figure 7-2: Figure 7-3: Figure 7-4:	Reduction of Pairwise Comparisons Via a Duality Approach
7 Figure 7-1: Figure 7-2: Figure 7-3: Figure 7-4:	Reduction of Pairwise Comparisons Via a Duality Approach
7 Figure 7-1: Figure 7-2: Figure 7-3: Figure 7-4: Figure 7-5:	Reduction of Pairwise Comparisons Via a Duality Approach
7 Figure 7-1: Figure 7-2: Figure 7-3: Figure 7-4: Figure 7-5:	Reduction of Pairwise Comparisons Via a Duality Approach
7 Figure 7-1: Figure 7-2: Figure 7-3: Figure 7-4: Figure 7-5:	Reduction of Pairwise Comparisons Via a Duality Approach
7 Figure 7-1: Figure 7-2: Figure 7-3: Figure 7-4: Figure 7-5: Figure 7-6:	Reduction of Pairwise Comparisons Via a Duality Approach
7 Figure 7-1: Figure 7-2: Figure 7-3: Figure 7-4: Figure 7-5: Figure 7-6:	Reduction of Pairwise Comparisons Via a Duality Approach

List of Figures

8	A Sensitivity Analysis Approach
	for MCDM Methods131
Figure 8-1:	Frequency of the time that the PT Critical
	Criterion is the Criterion with
	the Highest Weight
Figure 8-2:	Frequency of the time that the PT Critical
	Criterion is the Criterion with
	the Lowest Weight
Figure 8-3:	Frequency of the time that the PA Critical
	Criterion is the Criterion with
	the Highest Weight
Figure 8-4:	Frequency of the time that the PA Critical
	Criterion is the Criterion with
	the Lowest Weight
Figure 8-5:	Frequency of the time that the AT Critical
	Criterion is the Criterion with
	the Highest Weight
Figure 8-6:	Frequency of the time that the AT Critical
	Criterion is the Criterion with
	the Lowest Weight
Figure 8-7:	Frequency of the time that the AA Critical
	Criterion is the Criterion with
	the Highest Weight
Figure 8-8:	Frequency of the time that the AA Critical
	Criterion is the Criterion with
	the Lowest Weight
Figure 8-9:	Frequency of the time that the AT and PT
	Definitions point to the Same Criterion
Figure 8-10:	Frequency of the time that the AA and PA
	Definitions point to the Same Criterion
Figure 8-11:	Frequency of the time that the AT, PT, AA, and PA
-	Definitions point to the Same Criterion
	Under the WSM Method
Figure 8-12:	Rate that the AT Criterion is the one
0	with the Lowest Weight for Different Size
	Problems Under the WPM Method 154
9	Evaluation of Methods for Processing a
	Decision Matrix and Some Cases
	of Ranking Abnormalities 177

xvi	MCDM Methods: A Comparative Study, by E. Triantaphyllou
	WSM and the AHP
Figure 9-2:	Contradiction Rate (%) Between the
0	WSM and the Revised AHP
Figure 9-3:	Contradiction Rate (%) Between the
0	WSM and the WPM
Figure 9-4:	Rate of Change (%) of the Indication of the
0	Optimum Alternative When a Non-Optimum
	Alternative is Replaced by a Worse one.
	The AHP Case
Figure 9-5:	Rate of Change (%) of the indication of the
	Optimum Alternative When a Non-Optimum
	Alternative is Replaced by a Worse one.
	The Revised AHP Case 191
Figure 9-6.	Contradiction Rate (%) Between the WSM
rigure > 0.	and TOPSIS Method 196
Figure 9.7.	Rate of Change (%) of the Indication of the
1 igure <i>y</i> =7.	Ontimum Alternative When a Non-Ontimum
	Alternative is Replaced by a Worse one
	The TOPSIS Case 196
Figure 0 8.	Indication of the Best MCDM Method According
Figure 9-0.	to Different MCDM Methods
10	A Computational Evaluation of the Original
10	A Computational Evaluation of the Original and the Revised AHP201
10 Figure 10-1:	A Computational Evaluation of the Original and the Revised AHP
10 Figure 10-1:	A Computational Evaluation of the Original and the Revised AHP
10 Figure 10-1: Figure 10-2:	A Computational Evaluation of the Original and the Revised AHP
10 Figure 10-1: Figure 10-2:	A Computational Evaluation of the Original and the Revised AHP
10 Figure 10-1: Figure 10-2:	A Computational Evaluation of the Original and the Revised AHP
10 Figure 10-1: Figure 10-2: 11	A Computational Evaluation of the Original and the Revised AHP
10 Figure 10-1: Figure 10-2: 11	A Computational Evaluation of the Original and the Revised AHP
10 Figure 10-1: Figure 10-2: 11 Figure 11-1:	A Computational Evaluation of the Original and the Revised AHP
 10 Figure 10-1: Figure 10-2: 11 Figure 11-1: 	A Computational Evaluation of the Original and the Revised AHP
 10 <i>Figure 10-1:</i> <i>Figure 10-2:</i> 11 <i>Figure 11-1:</i> 	A Computational Evaluation of the Original and the Revised AHP
 10 <i>Figure 10-1:</i> <i>Figure 10-2:</i> 11 <i>Figure 11-1:</i> 	A Computational Evaluation of the Original and the Revised AHP
 10 Figure 10-1: Figure 10-2: 11 Figure 11-1: Figure 11-2: 	A Computational Evaluation of the Original and the Revised AHP
 10 <i>Figure 10-1:</i> <i>Figure 10-2:</i> 11 <i>Figure 11-1:</i> <i>Figure 11-2:</i> 	A Computational Evaluation of the Original and the Revised AHP
 10 <i>Figure 10-1:</i> <i>Figure 10-2:</i> 11 <i>Figure 11-1:</i> <i>Figure 11-2:</i> 	A Computational Evaluation of the Original and the Revised AHP
 10 <i>Figure 10-1:</i> <i>Figure 10-2:</i> 11 <i>Figure 11-1:</i> <i>Figure 11-2:</i> 	A Computational Evaluation of the Original and the Revised AHP
 10 Figure 10-1: Figure 10-2: 11 Figure 11-1: Figure 11-2: Figure 11-2: 	A Computational Evaluation of the Original and the Revised AHP
 10 <i>Figure 10-1:</i> <i>Figure 10-2:</i> 11 <i>Figure 11-1:</i> <i>Figure 11-2:</i> <i>Figure 11-3:</i> 	A Computational Evaluation of the Original and the Revised AHP

List of Figures	
-----------------	--

	Considered Together and in Pairs.	226
Figure 11-4.	Contradiction Rates on the Indication of	
1 igure 11 1.	Any Alternative When Alternatives are	
	Considered Together and in Pairs	
	The Ideal Mode (Revised) AHP Case	226
Figure 11-5:	Contradiction Rates on the indication of	
	Any Alternative When Alternatives are	
	Considered in Pairs.	
	The Original AHP Case	
Figure 11-6:	Contradiction Rates on the indication of	
0	Any Alternative When Alternatives are	
	Considered in Pairs.	
	The Ideal Mode AHP Case	
12	Fuzzy Sets and Their Operations	235
Figure 12-1:	Membership Functions for the Two Fuzzy	
0	Alternatives \hat{A}_1 and \hat{A}_2	
10		
13	Fuzzy Multi-Criteria Decision Making	241
13 Figure 13-1:	Fuzzy Multi-Criteria Decision Making Membership Functions of the Fuzzy Alternatives	241
13 Figure 13-1:	Fuzzy Multi-Criteria Decision Making Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-1 According	241
13 Figure 13-1:	Fuzzy Multi-Criteria Decision Making Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-1 According to the Fuzzy WSM Method	241 243
13 Figure 13-1: Figure 13-2:	Fuzzy Multi-Criteria Decision Making Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-1 According to the Fuzzy WSM Method Membership Functions of the Fuzzy Alternatives	241 243
13 Figure 13-1: Figure 13-2:	Fuzzy Multi-Criteria Decision Making Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-1 According to the Fuzzy WSM Method Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-2 According to the Fuzzy WDM Method	241 243
13 Figure 13-1: Figure 13-2:	Fuzzy Multi-Criteria Decision Making	241 243 244
13 Figure 13-1: Figure 13-2: Figure 13-3:	Fuzzy Multi-Criteria Decision Making Membership Functions of the Fuzzy Alternatives \hat{A}_1 , \hat{A}_2 , and \hat{A}_3 of Example 13-1 According to the Fuzzy WSM Method Membership Functions of the Fuzzy Alternatives \hat{A}_1 , \hat{A}_2 , and \hat{A}_3 of Example 13-2 According to the Fuzzy WPM Method Contradiction Rate <i>R11</i> When the Number of Euzzy Alternatives is Equal to 3	241 243 244 244
13 Figure 13-1: Figure 13-2: Figure 13-3: Figure 13-4:	Fuzzy Multi-Criteria Decision Making Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-1 According to the Fuzzy WSM Method Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-2 According to the Fuzzy WPM Method Contradiction Rate <i>R11</i> When the Number of Fuzzy Alternatives is Equal to 3	241 243 244 259
13 Figure 13-1: Figure 13-2: Figure 13-3: Figure 13-4:	Fuzzy Multi-Criteria Decision Making	241
13 Figure 13-1: Figure 13-2: Figure 13-3: Figure 13-4: Figure 13-5:	Fuzzy Multi-Criteria Decision Making	
13 Figure 13-1: Figure 13-2: Figure 13-3: Figure 13-4: Figure 13-5:	Fuzzy Multi-Criteria Decision Making Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-1 According to the Fuzzy WSM Method Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-2 According to the Fuzzy WPM Method Contradiction Rate <i>R11</i> When the Number of Fuzzy Alternatives is Equal to 3 Contradiction Rate <i>R11</i> When the Number of Fuzzy Alternatives is Equal to 21 Contradiction Rate <i>R21</i> When the Number of Fuzzy Alternatives is Equal to 21	
13 Figure 13-1: Figure 13-2: Figure 13-3: Figure 13-4: Figure 13-5: Figure 13-6:	Fuzzy Multi-Criteria Decision Making Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-1 According to the Fuzzy WSM Method Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-2 According to the Fuzzy WPM Method Contradiction Rate <i>R11</i> When the Number of Fuzzy Alternatives is Equal to 3 Contradiction Rate <i>R11</i> When the Number of Fuzzy Alternatives is Equal to 21 Contradiction Rate <i>R21</i> When the Number of Fuzzy Alternatives is Equal to 3	
13 Figure 13-1: Figure 13-2: Figure 13-3: Figure 13-4: Figure 13-5: Figure 13-6:	Fuzzy Multi-Criteria Decision Making	241 243 244 259 259 260 260
13 Figure 13-1: Figure 13-2: Figure 13-3: Figure 13-4: Figure 13-5: Figure 13-6: Figure 13-7:	Fuzzy Multi-Criteria Decision Making Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-1 According to the Fuzzy WSM Method Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-2 According to the Fuzzy WPM Method Contradiction Rate <i>R11</i> When the Number of Fuzzy Alternatives is Equal to 3 Contradiction Rate <i>R11</i> When the Number of Fuzzy Alternatives is Equal to 21 Contradiction Rate <i>R21</i> When the Number of Fuzzy Alternatives is Equal to 3 Contradiction Rate <i>R21</i> When the Number of Fuzzy Alternatives is Equal to 3 Contradiction Rate <i>R21</i> When the Number of Fuzzy Alternatives is Equal to 3 Contradiction Rate <i>R21</i> When the Number of Fuzzy Alternatives is Equal to 3 Contradiction Rate <i>R21</i> When the Number of Fuzzy Alternatives is Equal to 21 Contradiction Rate <i>R21</i> When the Number of	
13 Figure 13-1: Figure 13-2: Figure 13-3: Figure 13-4: Figure 13-5: Figure 13-6: Figure 13-7:	Fuzzy Multi-Criteria Decision Making Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-1 According to the Fuzzy WSM Method Membership Functions of the Fuzzy Alternatives \hat{A}_1, \hat{A}_2 , and \hat{A}_3 of Example 13-2 According to the Fuzzy WPM Method Contradiction Rate <i>R11</i> When the Number of Fuzzy Alternatives is Equal to 3 Contradiction Rate <i>R11</i> When the Number of Fuzzy Alternatives is Equal to 21 Contradiction Rate <i>R21</i> When the Number of Fuzzy Alternatives is Equal to 3 Contradiction Rate <i>R21</i> When the Number of Fuzzy Alternatives is Equal to 3 Contradiction Rate <i>R21</i> When the Number of Fuzzy Alternatives is Equal to 3 Contradiction Rate <i>R21</i> When the Number of Fuzzy Alternatives is Equal to 21 Contradiction Rate <i>R21</i> When the Number of Fuzzy Alternatives is Equal to 3	

xviii