Oblivious Routing in Wireless networks

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Outline of Presentation

Introduction
Network Model
Oblivious Algorithm
Analysis
Discussion
Routing: choose paths from sources to destinations
Edge congestion

\[ C_{\text{edge}} \]

maximum number of paths that use any edge

Node congestion

\[ C_{\text{node}} \]

maximum number of paths that use any node
Stretch = \frac{\text{Length of chosen path}}{\text{Length of shortest path}}

stretch = \frac{12}{8} = 1.5
Oblivious Routing

Each packet path choice is independent of other packet path choices
Path choices: $q_1, \ldots, q_k$

Probability of choosing a path: $Pr[q_i]$

$$\sum_{i=1}^{k} Pr[q_i] = 1$$
Benefits of oblivious routing:

• Distributed
• Needs no global coordination
• Appropriate for dynamic packet arrivals
Related Work

Valiant [SICOMP'82]:
First oblivious routing algorithms for permutations on butterfly and hypercube
Maggs, Meyer auf der Heide, Voecking, Westermann [FOCS’97]:

d-dimensional Grid: \( C_{edge} = O\left(d \cdot C^*_{edge} \log n\right) \)

Lower bound for oblivious routing:
\[ C_{edge} = \Omega\left(\frac{C^*_{edge} \log n}{d}\right) \]
Racke [FOCS’02]:

**Arbitrary Graphs:** \( C_{\text{edge}} = O\left( C_{\text{edge}}^* \log^3 n \right) \)

existential result

Azar et al. [STOC03]
Harrelson et al. [SPAA03]
Bienkowskies et al. [SPAA03]

constructive
Approach: Hierarchical clustering
At the lowest level every node is a cluster
Pick random node
Pick random node
Pick random node
Pick random node
Pick random node
Pick random node
Pick random node
Problem: Big stretch

Adjacent nodes may follow long paths
An Impossibility Result

Stretch and congestion cannot be minimized simultaneously in arbitrary graphs
Example graph:

Each path has length $\Theta(\sqrt{n})$

$\sqrt{n}$ paths

$n$ nodes

Source of $\sqrt{n}$ packets

Destination of all packets

Length 1
Stretch = 1

Edge congestion = $\sqrt{n}$

$\sqrt{n}$ packets in one path
Stretch = $\sqrt{n}$

Edge congestion = 1

1 packet per path
Contribution

Busch, Magdon-Ismail, Xi [SPAA 2005]:

Oblivious algorithm for special graphs embedded in the 2-dimensional plane

**Constant stretch**

\[ \text{stretch} = O(1) \]

**Small congestion**

\[ C_{\text{node}} = O(C_{\text{node}}^* \log n) \]

\[ C_{\text{edge}} = O(\Delta \cdot C_{\text{edge}}^* \log n) \]
Embeddings in wide, closed-curved areas
Our algorithm is appropriate for various wireless network topologies.

Transmission radius
Basic Idea

source  destination
Pick a random intermediate node
Construct path through intermediate node
Previous results for Grids:

Busch, Magdon-Ismail, Xi [IPDPS’05]

\[ C_{\text{edge}} = O(d \cdot C_{\text{edge}}^* \log n) \]

\[ \text{Stretch} = O(d^2) \]

For \( d=2 \), a similar result given by C. Scheideler
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Network $G$  Surrounding area $A$
Perpendicular bisector
\[ \gamma(x, y) = \frac{s}{\|x, y\|} \]
Area wideness: \( \gamma = \min_{x,y \in A}(\gamma(x,y)) \)
Coverage Radius $R$:
maximum distance from a space point to the closest node
For all pair of nodes there exist $\alpha, \beta$:

$$\alpha \leq \frac{\text{dist}_G(u,v)}{\|u,v\|} \leq \beta$$

Euclidian distance: $\|u,v\|$

Shortest path length: $\text{dist}_G(u,v)$

$$\frac{\text{dist}_G(u,v)}{\|u,v\|} = \frac{8}{5} = 1.6$$
Consequences of

\[
\alpha \leq \frac{\text{dist}_G(u,v)}{\|u,v\|} \leq \beta
\]

Max Euclidian distance between adjacent nodes

\[
\|u,v\| \leq \frac{1}{\alpha}
\]

(max transmission radius in wireless networks)
Consequences of \( \alpha \leq \frac{\text{dist}_G(u,v)}{|u,v|} \leq \beta \)

Min Euclidian Distance between any pair of nodes: \( |u,v| \geq \frac{1}{\beta} \)

\( O((\beta r)^2) \) nodes
Good Network embeddings:

Small $\alpha, \beta, R$ and large $\gamma$

Suppose they are constants
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Every pair of nodes is assigned a default path.

Examples:

• Shortest paths

• Geographic routing paths (GPSR)
The algorithm
Perpendicular bisector
Pick random space point $y$
Find closest node to point $y$
Connect intermediate node $w$ to source and destination.
Consider an arbitrary set of packets:

\[ \Pi = \{ \pi_1, \ldots, \pi_N \} \]

Suppose the oblivious algorithm gives paths:

\[ P = \{ p_1, \ldots, p_N \} \]
We will show:

\[
\text{stretch} = O(1)
\]

\[
C_{\text{node}} = O(C_{\text{node}}^* \cdot \log n)
\]

optimal congestion
Theorem: \( \text{stretch} = O(1) \)

Proof: Consider an arbitrary path \( p \in P \) and show that:

\[
\text{stretch}(p) = O(1)
\]
\[ \text{stretch}(p) = \frac{\text{length}(p)}{\text{dist}_G(s,t)} = \frac{\text{length}(q_1) + \text{length}(q_2)}{\text{dist}_G(s,t)} \]
\[ \text{stretch}(p) = \frac{\text{length}(q_1) + \text{length}(q_2)}{\text{dist}_G(s,t)} \]

when default paths are shortest paths

\[ \text{stretch}(p) = \frac{\text{dist}_G(s,w) + \text{dist}_G(w,t)}{\text{dist}_G(s,t)} \]

we show this is constant
\[ \text{dist}_G(s,w) \leq \beta \cdot \|s,w\| \leq \beta \cdot (\|s,y\| + R) \leq \beta \cdot (\|s,t\| + R) \]

Similarly:

\[ \text{dist}_G(w,t) \leq \beta \cdot (\|s,t\| + R) \]
\[ \text{dist}_G(s,t) \geq \alpha \cdot \|s,t\| \]

\[ \alpha \leq \frac{\text{dist}_G(s,t)}{\|s,t\|} \leq \beta \]
\[
\text{stretch}(p) = \frac{\text{dist}_G(s,w) + \text{dist}_G(w,t)}{\text{dist}_G(s,t)} \leq \frac{2\beta \cdot (\|s,t\| + R)}{\alpha \cdot \|s,t\|}
\]

For \(\alpha, \beta, R\) constants:

\[
\text{stretch}(p) = O(1)
\]

End of Proof
Theorem: Expected case:

\[ C = O(\mathcal{C}^* \cdot \log n) \]

Proof: Consider some arbitrary node \( V \) and estimate congestion on \( V \).
Deviation of default paths:

maximum distance from geodesic

\[ \text{deviation} = \max \left( \text{deviation} \left( q_i \right) \right) \]
Consider some path from $s$ to $t$
the use of $v$ depends on the choice of space point $y$
another choice
If you choose node \( w \) in the cone, the respective path may use \( V \).
If you choose node $w$ outside the cone the respective path does not use $v$. 

$\text{deviation}$ 

$\text{cone affecting}$ 

$S$ 

$t$ 

$w$ 

$v$
Segment of space points affecting $V$

deviation

cone affecting $V$
Probability of using node $v$ : $\Pr[v] \leq \frac{l_1}{l_2}$

deviation$(Q)$

cone affecting $v$
It can be shown that:

\[ \Pr[v] \leq \frac{\ell_1}{\ell_2} \leq \frac{k_1}{\gamma} \left( \frac{R}{\|s,t\|} + \frac{\text{deviation}}{\|s,v\|} \right) \]
\[ \|s,v\| \leq \|s,t\| + R + \text{deviation}(Q) \]

for simplicity
assume:  \[ \|s,v\| \leq \|s,t\| \]
\[ \Pr[v] \leq \frac{k_1}{\gamma} \left( \frac{R}{\|s,t\|} + \frac{\text{deviation}}{\|s,v\|} \right) \]

\[ \|s,v\| \leq \|s,t\| \]

\[ \Pr[v] \leq \frac{k_1(R + \text{deviation})}{\gamma \|s,v\|} \]

\[ \gamma, R, \text{deviation}: \text{constants} \]

\[ \Pr[v] \leq \frac{k_2}{\|s,v\|} \]
Divide area \( A \) into concentric circles

\[ r_i = \frac{2^i}{\beta} \]
Max Euclidian distance between any two nodes = \frac{n}{\alpha}

Longest path has at most \( n \) nodes

\[ \| u_i , u_{i+1} \| \leq \frac{1}{\alpha} \]
\[ r_i = \frac{2^i}{\beta} \]

Maximum ring radius

\[ A \log_{\frac{\beta n}{\alpha}} r \leq \frac{n}{\alpha} \]
\[ N_i = \text{number of packets that can affect } \nu \]
\[ C_i = \text{number of paths that use } \nu \]

We will bound
\[ \Pr[v] \leq \frac{k_2}{\|s,v\|} \leq \frac{k_2}{r_{i-1}} \]
Expected congestion: $E[C_i] \leq N_i \cdot Pr[v] \leq \frac{k_2 N_i}{r_{i-1}}$
We have proven

\[ E[C_i] = O\left( \frac{N_i}{r_{i-1}} \right) \]

\[ r_{i+1} = 4 \cdot r_{i-1} \]

\[ C^* = \Omega\left( \frac{N_i}{r_{i+1}} \right) \]

we prove next
we showed earlier
Similarly, each packet that affects $v$ traverses distance at least $r_{i-1}$.
\[ \alpha \leq \frac{\text{dist}_G(s, t)}{\|s, t\|} \leq \beta \]

\[ \|s, t\| \geq r_{i-1} \]

\[ \text{dist}_G(s, t) \geq \alpha r_{i-1} \]
Total number of nodes used \( \geq N_i \cdot \alpha r_{i-1} \)
Average node utilization \[ \geq \frac{N_i \cdot \alpha r_{i-1}}{\# \text{nodes in area } X} \]
#nodes in area $X = O((\beta r_{i+1})^2)$
Average node utilization \( \geq \frac{N_i \cdot \alpha r_{i-1}}{O((\beta r_{i+1})^2)} = \Omega\left(\frac{N_i}{r_{i+1}}\right) \)

\[ C^* \geq \text{average node utilization} \]

\[ C^* = \Omega\left(\frac{N_i}{r_{i+1}}\right) \]
We have proven:

\[ E[C_i] = O\left(\frac{N_i}{r_{i-1}}\right) \]

\[ r_{i+1} = 4 \cdot r_{i-1} \]

\[ C^* = \Omega\left(\frac{N_i}{r_{i+1}}\right) \]

\[ E[C_i] = O(C^*) \]
Considering all the rings:

$$E(C) = \sum_{i=0}^{\alpha} E[C_i]$$

$$= O\left(C^* \cdot \log\frac{\beta n}{\alpha}\right)$$

$$= O(C^* \cdot \log n)$$

End of Proof
Recap

We presented a simple oblivious algorithm which has:

**Constant stretch**

\[ \text{stretch} = O(1) \]

**Small congestion**

\[ C_{\text{node}} = O(C^*_{\text{node}} \log n) \]
\[ C_{\text{edge}} = O(\Delta \cdot C^*_{\text{edge}} \log n) \]

when the parameters of the Euclidian embedding are constants
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Holes
Arbitrary closed shapes

there is no $\gamma$